DECLARATION

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DEDICATION

To my family and Guadalupe, whose patience and support have enabled me to write this thesis.

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ABSTRACT

This thesis presents a simplified methodology for the assessment of Catalan double curvature masonry vaults. The methodology consists on discretize the vault in a series of arches capable of representing the structural behavior of the vault. This network of hanging cables catenaries that represents the thrust lines of the structure in an inverted model. With the application of the limit theorems of plasticity and forcing an optimization process, the methodology can be used to assess the safety of the structure and estimate the ultimate load.

Crack mapping and elastic analysis performed in SAP were used to better understand the behavior of the vault, and make an accurate model capable of represent the structural behavior of the vault and make a good assessment on the vaults safety.

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RESUMEN

En la tesis se presenta una metodología simplificada para la evaluación estructural de bóvedas Catalanas de doble curvatura. La metodología consiste en discretizar la bóveda en una serie de arcos capaces de representar el comportamiento estructural de la bóveda. Esta red de catenarias colgantes que representan la línea de empujes de la estructura en un modelo invertido. Con la aplicación de los teoremas limite de plasticidad y forzando un proceso de optimización, la metodología puede ser usada para la evaluación de la seguridad de la estructura y estimar la carga ultima que puede resistir antes del colapso.

La identificación de fisuras y análisis elásticos realizados en un programa de elementos finitos fueron usados para un mejor entendimiento del comportamiento estructural y construir modelos capaces de representar el comportamiento estructural de las bóvedas y hacer una buena evaluación en términos de seguridad.

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1. INTRODUCTION

The assessment of the stability of masonry constructions has challenged builders and scientists from ancient times. In the past, the construction of buildings and bridges was based on geometrical rules. One of the first rational approaches to the problem was found in the analogy between the shape of masonry arches in equilibrium and that of hanging cables working in tension. The analogy or catenary principle is known from the 17th century and was first presented by Robert Hooke in a famous anagram.

In particular, the so-called safe uniqueness theorem leads to envisage equilibrium line solutions associated with unstable ductile mechanisms, where each contact point between the equilibrium line and the boundary of the structure causes a plastic hinge, and a certain number of plastic hinges causes the collapse of the structure.

The application of these formulations in structures like arches or skeletal systems has been normally carried out using graphical or analytical methods. In practice, these approaches can be only applied to simple structures. The extension to more complex structures requires computational facilities. (O'Dwyer 1999) attempted to extend this approach to shell structures such as domes and vaults decomposed in discrete arches in equilibrium. The computational formulation of O'Dwyer also includes an optimization process to determine the ultimate load of the structure.

This thesis presents a simplified methodology for the assessment of Catalan double curvature masonry vaults. The methodology consists on discretize the vault in a series of arches capable of representing the structural behavior of the vault. This network of hanging cables catenaries that represents the thrust lines of the structure in an inverted model. With the application of the limit theorems of plasticity and forcing an optimization process, the methodology can be used to assess the safety of the structure and estimate the ultimate load

Crack mapping and elastic analysis performed in SAP were used to better understand the behavior of the vault, and make an accurate model capable of represent the structural behavior of the vault and make a good assessment on the vaults safety.

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2. THE CATALAN VAULT

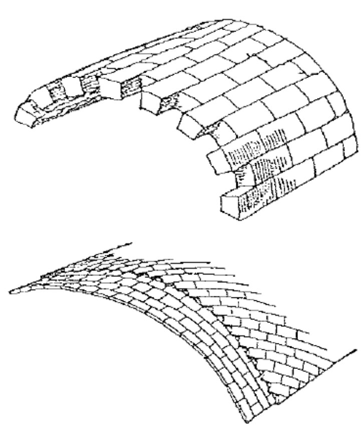
2.1 Introduction

Catalonian vault, which is also known as Guastavino or timbrel vaulting, are vaults made with bricks and mortar. Their singularity comes from their construction: Is a system of interlocking thin terracotta tiles and layers of mortar forming one or several sheets.

During their construction the fixing of the bricks is achieved by the adhesion of quick settling mortar with the previous arches or rings already finished. There are no arch centres but 'forms' are indeed used (light trusses), with the aim of controlling the geometry of the vault, in particular when this acquires certain dimensions. See *Figure 1*

This type of construction uses little or no steel. Due to its self-supporting capacity combined with the use of quick-setting mortar, greatly reduces the time and cost of construction compared to conventional masonry vaulting. The tiles and mortar form a homogeneous structure, infallible even to accidental penetrations.

The timbrel vaults can be built with very limited thickness. The typical is that there are 2 sheets (about 10 centimeters in total, including an intermediate layer of mortar and the covers), but they can be found with one sheet (about 5 cm. The slenderness ratio, the relationship between the radius of curvature and span is found around one hundred, but there are some more slender. They can also span large spaces, as in the case of the vault above the crossing of the cathedral of Saint John the Divine in New York, which with a 33 m span is the biggest to nave been built. (Huerta, 2004)



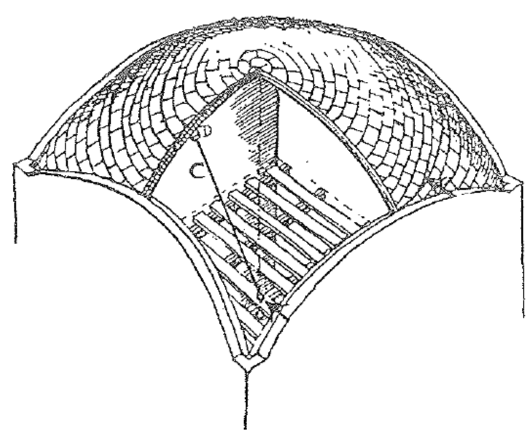


Figure 1: Construction of timbrel vaults (left) comparison between a timbrel vault and a stone voussoir vault; (right) construction without centering of a timbrel dome (Moya, 1957)

The use of Portland cement in the hook allowed them to be used as covers without the need of an upper roof or any other method of waterproofing. In Catalonia at the end of the nineteenth century and the beginning of the twentieth they almost became a national symbol. Rafael Guastavino exported them to America at the end of the nineteenth century, and there he bestowed on them a dignity which they had probably never had. "Guastavino's vaults" were built in some of the most important buildings in the east area of the United States between 1890 and 1900.

2.2 History Background early history

The history background about this technique and its dissemination throughout the world is discussed by Dietrich Neumann (Neumann, 1999). Here is described that predecessors to this vaulting technique seem to have existed in ancient Rome as part of certain barrel-vault constructions. Although they were not realized to their full potential, they reveal a similar desire by the Roman builders to minimize the depth of their vaults of brick and cement. Recently discovered documents prove that a more fully developed laminated vaulting technique was specified and discussed in the late Middle Ages in Spain. In 1382 the architect Pierre le Ceremonieux reported to King Merino about his work at the Royal Palace in Valencia in the Spanish region. The first one to analyze the technique in an architectural treatise seems to have been the Spanish monk Lorenzo de San Nicolas, who published his Arte y uso de Arquitectura in Madrid in 1663. Since then, the technique was illustrated quite regularly in architectural handbooks in France throughout the eighteenth century, both for its fireproof qualities and its superior construction. Among the prominent theorists who mention the method are the Comte d'Espie's Maniere de rendre toutes sortes d'edifices incombustibles in 1754 and Jean-François Blondel's and Pierre Patte's Cours d'Architecture of 1777. Jean-Baptiste Rondelet (1734-1829) showed the laminated vaulting technique in his Traite Theorique et pratique de l'art de Batir of 1802. Since Rondelet taught at both the Ecole Polytechnique and the Ecole des Beaux Arts, we can assume a certain degree of dissemination of the idea. All of these descriptions show the vault's application as an entirely internal system that had little influence on the spatial layout or formal language of a building. The accelerated growth of cities in the second half of the nineteenth century increased the need for fireproof construction. It is in this context that the thin brick vaults were used again with growing frequency. It is quite likely that their appreciation and rediscovery was furthered in northeastern Spain by the belief that they were a particularly Catalan invention. The growing movement towards Catalan independence went hand in hand with efforts to revive Catalan traditions in all aspects of daily life.

2.3 Rafael Guastavino and the Catalan vaults

Spanish architect Rafael Guastavino was born in Valencia on March 1, 1842. After studying at the Escuela de Maestros de Obras de Barcelona, he began designing and constructing major buildings in Spain with a system he called “cohesive construction.”

This system was described by Guastavino in a paper titled, “Cohesive Construction. Its Past, Its Present, Its Future” and was presented to the Congress of Architects in connection with the Columbian Exposition in Chicago on August 5, 1893 and later published in American Architect and Building News. In this paper, Guastavino describes cohesive construction as a method of building that relies on the adhesion of materials as opposed to the friction of the more commonly used gravity system. (Huerta, 2004)

Guastavino further clarifies the differences between the two systems in his Essay on the History and Theory of Cohesive Construction, Applied Especially to the Timbrel Vault. His illustrations clearly show that in a vault of single thickness with only one vertical joint, it is gravity alone that keeps the elements in place. According to Guastavino, adding a second layer with breaking joints and mortar significantly decreases the weight of a similar vault and provides greater load capacity due to the cohesion of the materials. See *Figure 2*

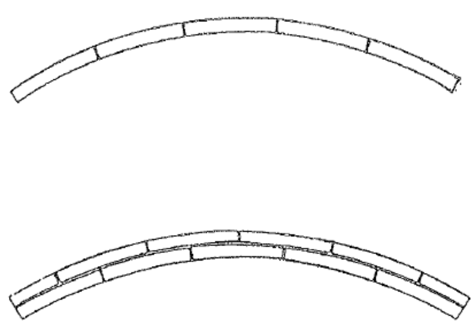


Figure 2 : Comparison between a 'mechanical' arch (above) and a 'cohesive arch'

Before emigrating to the U.S., Rafael Guastavino employed laminated vaults in a sober and pragmatic style in a number of industrial buildings in Barcelona, most significantly in his factory for the Batllo Brothers (today called the Clock Building) of 1869-1875. In 1876, he exhibited the Catalan vault at the Philadelphia World's Fair. Encouraged by his success, he and his son Rafael Jr. brought the technology to the United States five years later. The Guastavino’s did not stay in Barcelona long enough to witness the stylistic changes in and around the city from the late 1880’s onwards. The deliberate strangeness of the playful architecture at the 1888 World's Fair in Barcelona, with its exuberant and creative ornamentation and highly colorful materials, paved the way for the Catalan Modernism at the turn of the century.

2.4 Catalan Vaults and Modernism

Antonio Gaudi and his contemporaries Lluis Domenech I Montaner and Josep Puig I Cadafalch were among the many architects who integrated the Catalan vaults into a number of different building types and explored their potential further. It is in these buildings, considered outstandingly and uniquely Catalan, that the vaulting method not only reached its full potential for the first time but also gained meaning and political symbolism.

Figure 3: Muelle de descarga, Celler de Gandesa Tarragona by Cesar Martinelli.

It seems that the use of Catalan vaults as an expressive medium receded under the spread of the formal vocabulary of the Modern movement, which progressed hand-in-hand with less labor-intensive construction methods. There are a few exceptions, though. The Swiss architect Le Corbusier had long had an interest in the indigenous architecture of the Mediterranean. On a trip to Barcelona in 1928 he sketched Gaudi's school at the Sagrada Familia and was one of the few Modernists to recognize Gaudi's genius early on. He used Catalan vaults as permanent formwork for the concrete roofs of the Maison Jaoul in Paris in 1955. Their use as permanent formwork in the flat, barrel-vault application, though, was so different and formally so much simpler that a direct influence from Gaudi does not seem likely. After the enormous success of the formal language Corbusier had helped to formulate in the 1920’s, contemporary critics noted with some astonishment his return to craft techniques and the evocation of indigenous forms. The British architect James Stirling noticed that the Maison Jaoul was "essentially exotic and anti-urban," contained "cave-like interior volumes," and was "anti-mechanistic, traditionalist, earth hugging." This passage illuminates well the difficulties of Western architects in accepting anything that did not fit into the narrowly defined ideals of progress and modernity. In a rather curious comment, Stirling remarked that the house "was created for a status quo as present and actual as the migrant Algerian laborer which worked on it." The Algerian laborer could have indeed known Catalan vaults, since they had been tried out successfully in Algeria in 1947.

Shell structures using reinforced concrete had been developed during the 1920’s in Germany. It is not clear whether Walter Bauersfeld, who is usually credited with this development, knew of the earlier brick shells. However, the Spanish architects Eduardo Torroja (1899-1961) and Felix Candela (b. 1910) in the 1950’s employed reinforced concrete shell structures for the development of a new expressive architecture and clearly knew about and had been inspired by Catalan vaulting techniques. In his book Philosophy of Structures (1958) Torroja observed that the Catalonian vault, as indigenous to the country where it originated as are o live trees and groves. It is so marvelous in its realization, that theory is taxed to explain and to evaluate its resistant phenomenon, which was so easily and subconsciously sensed by builders long since buried many centuries ago in the same earth from which they made these remarkable bricks.

2.5 The Vaults of Eladio Dieste

While Torroja's and Candela's concrete shells had translated the central ideas behind the laminated brick structures into another material, others continued to develop the Catalan vaults them-selves. The Uruguayan architect Eladio Dieste (b. 1917) applied the central principles of the Catalan vault and improved it structurally but insisted that his inspiration came from the thin concrete shells of architects and engineers like Torroja and Candela. His most significant improvement was the use of steel reinforcement rods and tie-bars in conjunction with double-curvature brick shells, which increased the possible span of each unit. In both cross and longitudinal section, the catenary curve proved to be most statically resistant. In his writings Dieste placed the technique in the context of the global building market and the pressure on poorer countries to adopt industrialized building methods.

What is less widely known is that brick can resist some stresses better than some of the best concrete, and that concrete and mortar cannot equal baked earth in lightness. We have been able to produce structures that because of their light weight would be impossible in reinforced concrete. We have produced shells of double curvature to which the variable longitudinal undulations give the necessary rigidity to face flexion and elastic instability. All of the cross sections are catenary curves, and given its light weight, the shell is under very small stresses. These techniques have proved an economical and rational substitute for prefabricated concrete and steel systems. We have produced large spans with a high building speed and a relatively small work force. Even in the most artistic applications, like the churches that we have built, the costs have been absurdly low. There is nothing more noble and elegant from an intellectual viewpoint than this: to resist through form.

Dieste attributed the dwindling success of the curved brick shells to the "tyranny of the drawing board"; if designs were too difficult to draw, they were not built. Among Dieste's most stunning creations are a church in Atlantida, Uruguay (1 958), with undulating walls and ceiling based on a principle similar to that of Gaudi' school at the Sagrada Familia. Equally important is an early1960’s warehouse in Montevideo that is spanned by double-curved laminated shell structures similar to those in Lluis Muncunill's 1919 textile factory in Terrassa. Dieste's own house i n Montevideo (1962) employs the simple, flat, barrel vaults of a laminated bricks hell that Corbusier had used a few years earlier in the Maison Jaoul.

2.6 The Cuban Art Schools

Perhaps the most spectacular attempt at using the Catalan vaulting system as a political and cultural symbol occurred in Cuba after the revolution. In 1959 Fidel Castro and Ché Guevara decided to build a group of new art schools, Las Escuelas Nacionales de Arte, in an area that had previously been a prestigious Cuban golf club. Castro personally commissioned the young architect Ricardo Porro (b. 1925) to build them. Porro had studied in Havana and Paris and admired Le Corbusier's expressive late work. Together with two Italian friends, Vittorio Garatti and Roberto Gottardi, Porro began the design in 1961, just days after the Bay of Pigs incident. Porro assumed the general leadership and designed the schools for modern dance and plastic arts, Roberto Gottardi designed the school for dramatic arts, and Vittorio Garatti that for music and ballet. With Castro's approval Porro chose the Catalan vaults as the primary structural system. It was affordable and allowed for a highly unusual expressiveness that could be seen as a fitting symbol for the young Cuban revolution. All of the buildings exploited the potential of the material by forming clusters of domes and sequences of barrel vaults. The way in which these buildings took on urbanistic and spatial challenges and simultaneously responded to the surrounding landscape makes this complex one of the most important architectural creations of the early 1960’s. The success of the whole project rested on a quiet mason, Gumersindo, from Barcelona, whose father had worked for Antonio Gaudi. Gumersindo's sample vaults helped to overcome the authorities' initial skepticism. At first, progress on the schools was fast, and then slowed by political developments, the missile crisis in 1962, and the need to bring workers to other building sites. Finally, in various stages of completion, the schools were declared finished in 1965. Growing Russian influence simultaneously led to a stronger emphasis on prefabrication and a return to so-called rationalist principles. In 1968 the Catalan vaults were criticized as being individualist, monumental and authoritarian, instead of "scientific," "flexible”, and “efficient."

2.7 Conclusions

Laminated vaults have played different roles in response to different architectural cultures. The sheer number of buildings executed with this method in the U.S. between the 1890's and the 1930's seems to indicate a degree of success unequalled anywhere else. Their formal and expressive potential for double-curves, undulating walls, and rolling ceilings, however, remained largely unexplored.

In Catalonia, Cuba, and Uruguay, their unusual structural qualities helped to create a different formal language and liberated architects in search of a new, politically meaningful, modern architecture. Here it stood for the political and cultural in dependence of Catalonia. It is impressive how this cheap, labor-intensive low-tech building could overcome the highly industrialized building production of the time.

The recent rediscovery of the Catalan vault by architectural historians represents a shift towards a broader and more balanced history in which materials and their meaning will play a greater role. The Catalan vault in particular offers proof that Louis Sullivan's interpretation of building materials as potential carriers of poetic and social significance is as valid today as it was a century ago.

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3. ANALYSIS BACKGROUND

Limit analysis investigates the safety of masonry structures based on equilibrium and geometry rather than the strength of the material (Heyman, 1995). See *Figure 5*. Equilibrium methods, such as graphical analysis, rely on equations of equilibrium to find the forces within a structure. Graphical analysis holds the distinction of being one method used historically in the design of Catalan vaults and thus becomes a promising means of analyzing these structures today. Intuitive, quick to execute and easily adaptable to changes such as support movements and multiple load conditions, graphical analysis proposes forces within a Catalan vault without taking the material properties into consideration.

Elastic analysis methods, such as finite element analysis, utilize constitutive relations, boundary conditions, and imposed deformations to find the forces within the vault. In order to analyze a structure using finite element analysis, material properties of the structure such as the modulus of elasticity are needed. Elastic solutions are highly sensitive to small movements of the supports, which are inevitable, and make the exact stress state of a structure unknowable. Additionally, movements of the supports can induce cracks in brittle masonry shells, and cracks render the use of linear elastic finite element analysis inadequate for understanding the structure after cracking.

Besides a reliable analysis technique, another key element to the accurate assessment of the Catalan vault safety is an understanding of their behavior. Familiarity with the behavior of these structures and some common types of distress will aid an engineer in assessment by helping them to focus on the most critical aspects and avoid senseless investigations of nonthreatening characteristics.

This thesis will focus on the behavior of this vaulting system through the study of pathologies and structural analysis of two study cases. Instances of past structural problems will be considered for what they reveal about the behavior of masonry tile vaults. Both elastic and equilibrium analysis methods will be considered for their ability to assess the safety of these structures. Finally, the case studies will be used to propose a simplified methodology for the assessments of these structures and the understanding of the structural behavior of the Catalan vaults.

3.1 Scope of Research

This thesis aims to serve practicing architects and engineers as a guide in the assessment and restoration of Catalan vaults. The first step is to investigate the behavior of these structures by identifying common structural problems and considering the results of structural analyses. Then, general guidelines for making an accurate assessment of the safety of Catalan vaulted structures can be recommended.

One important consideration is that the recommendations presented herein be straightforward, practical, and reproducible. Of course, due to their complicated interactions with surrounding structural components and frequently cracked states, the complexity of these structures in real life is considerable.

While sophisticated analyses methods may seem well suited for the assessment of these structures, this research will demonstrate that less complicated analyses are nonetheless useful and valuable tools. In the interest of protecting these structures against unnecessary retrofits and demolition, a simplified methodology to assess these structures may be more powerful than an expensive and arduous structural analysis program.

3.2 The Analysis of Masonry Vaults

Before the advent of structural mechanics, builders used rules of proportion to design their structures Modern engineers design structures using the concepts of stress and strain from structural mechanics (based on elastic analysis), whose origins date back to Galileo in 1638. Robert Hooke published an anagram in 1675 which translates to "as hangs the flexible line, so but inverted will stand the rigid arch," and this theory is essentially the basis of equilibrium analysis (Heyman 1995). See *Figure 4.*

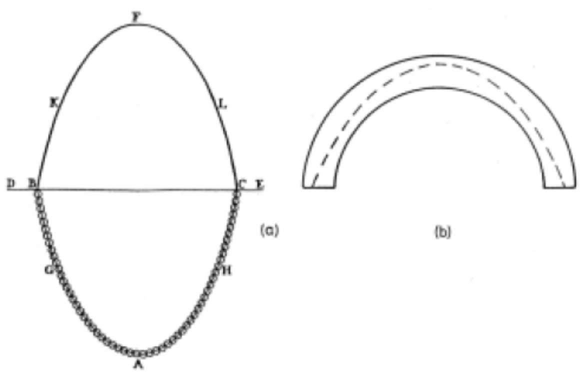


Figure 4: Idea of Hooke about the relationship between the arch and the centenary (Poletti, 1748)

Elastic analysis methods and equilibrium analysis methods did not necessarily develop at the same rate, but they have coexisted for several centuries. Within the field of unreinforced masonry design and analysis today, these two methods still coexist though not quite peacefully, and there is still much debate over which is the "correct" method for the analysis of masonry structures.

Dunn (1908) indicates that this rivalry existed even a century ago when he addresses the fallacy of applying elastic theory to masonry domes and arches. He notes that engineers of that time used elastic theory to design metallic bridges, but that elastic theory is not applicable to masonry because of the nonlinear behavior of the material and its lack of tensile capacity

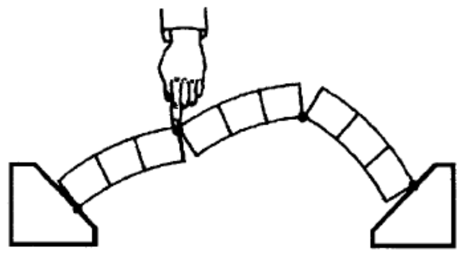


Figure 5: Failure of model arch with rigid voussoirs; failure load is independent of voussoir strength. (O'Dwyer, 1998)

Block et al. (2005) considered two masonry arches of different thickness analyzed with finite element analysis and thrust line analysis. The finite element results for the two models show little difference, but the thrust line analyses show that while one of the vaults is stable, the other would not stand as it is too thin to contain the thrust line. See *Figure 6*.

The results of this study make evident that linear elastic finite element analysis is unable to tell the analyst anything about the stability of the arch while the thrust line analysis demonstrates this very clearly, making it useful for the assessment of historic masonry structures. Furthermore, the authors note that while non-linear finite element analysis can be useful for crack propagation with the correct material properties, these properties are hard to predict. Such analyses can be time consuming, but limit analysis provides fast and reliable means to predict the collapse mechanism.

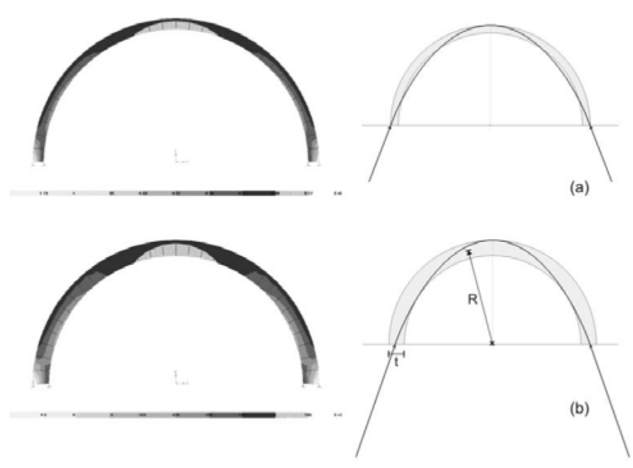


Figure 6: Elastic finite element analysis (left) versus limit state analysis (right) for arches with t/R ratios of (a) 0.08 and (b) 0.16. (Block, 2005)

3.3 Graphical Analysis

Graphical analysis investigates the equilibrium of a structure and can be applied to masonry structures in particular. Some earlier graphical methods, such as the development of force polygons (See *Figure 7)***,** descriptive geometry and projective geometry, are the precursors of Culmann's graphical statics method.

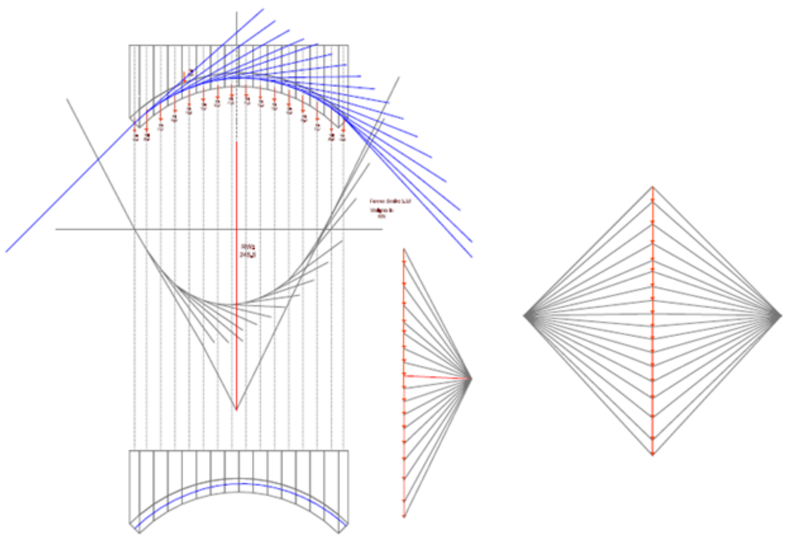


Figure 7: Graphic Statics applied to assess the safety of an arch

Traditional equilibrium analyses of masonry structures are still considered valid today since they satisfy the rules of a more modem development known as plastic theory. (Heyman, 1995) proposes that if a solution of compressive forces in equilibrium can be found to lie within the masonry then the structure is safe. See *Figure 8* to see how Gaudi used graphical statics to design their projects.

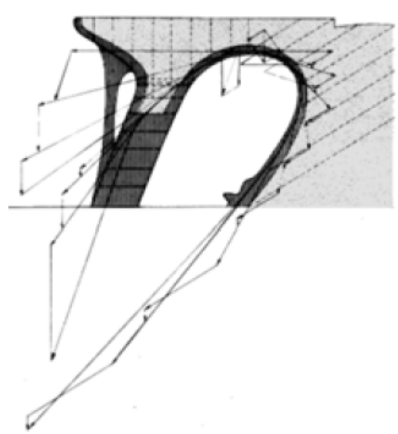


Figure 8: Gaudí's graphical design for the columns and retaining wall of the Park Güell

Heyman describes the pathologies of masonry arches and domes, which due to their brittle nature and little to no tensile capacity, must crack in response to support movements. For an arch, such cracks do not necessarily indicate that the structure is unstable, but only that it is now statically determinate (i.e. the path of the thrust line is known). Collapse occurs when the thrust line no longer can fit within the thickness of the arch. The vault, similarly, carries loads through forces within its thickness, but in two directions rather than one like the arch. (Heyman, 1995)

Block and Ochsendorf (2007) have developed a methodology called thrust network analysis that is capable of finding funicular solutions for three-dimensional shell structures. Using their methodology, finding a compression-only thrust network that fits within a three dimensional vault shape demonstrates that the structure is safe based on limit analysis. This method holds great potential to establish three-dimensional equilibrium solutions for masonry tile shell structures, but its application to Catalan vaulting is beyond the scope of this thesis.

3.4 Heyman's analysis

The position of the line of thrust is indeterminate to the third degree. For any arch there exists a family of possible lines of thrust. To draw the true line of thrust one must know the magnitude of the horizontal force at the abutments and the position of the line of thrust at two points. Traditionally, designers were satisfied that an arch was stable when any line of thrust could be found which was in equilibrium with the loads on the arch and which was contained within the arch ring. Heyman justified this assumption by demonstrating that the safe theorem of plasticity can be applied to arcuate masonry structures (Heyman, 1982). The assumptions about the behavior of masonry which Heyman made in order to apply the safe theorem are:

1. Masonry has no tensile strength.
2. Friction between the voussoirs is sufficient to prevent failure due to sliding of one voussoir relative to another.
3. Masonry has infinite compressive strength.

These are the classical assumptions which have been used since Couplet’s and Coulomb's early analyses. The first assumption is safe and the second assumption has been shown to be correct in practice. The third assumption is unsafe. However, because most arches fail as mechanisms it is not unreasonable, and the effects of local failure of the masonry at a hinge can be allowed for (Heyman, 1982).

Heyman's safe theorem as applied to arcuate masonry structures states: if a system of forces can be found which is in equilibrium with the loads on the structure and which is contained within the masonry envelope then the structure will carry the loads, although the forces in the structure may not be those postulated. (Heyman, 1977)

Once any system of forces can be found which lies within the structure and which is in equilibrium with the external loads acting on the structure then the structure is safe.

Just as the safe, lower bound, theorem of plastic analysis can be used to verify the safety of an arch, similarly a mechanism analysis can be used to calculate an upper bound on the arch's collapse load. The collapse mechanism for an arch rib is by the formation of four hinges which transform the arch into a four bar mechanism.

Calculating the imposed load required to cause any given mechanism to form is a task that will be performed by the spreadsheet presented in Appendix A, which allow an exhaustive search of all possible mechanisms to be undertaken.

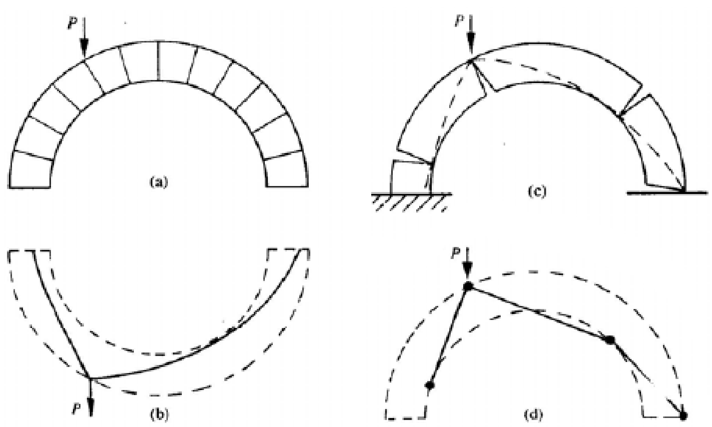


Figure 9: Collapse of a semicircular masonry arch under a point load (Heyman, 1995)

3.5 Analysis of masonry vaults

The analysis of masonry structures becomes more complex when three-dimensional force systems are considered. The mechanism method, which serves well for the linear arch, is less useful for masonry shells. The mechanism method gives an upper bound on the collapse load. The results of a mechanism analysis will be an accurate reflection of the true collapse load only if the critical failure mechanism can be identified.

The critical failure mechanism is difficult to identify if either the structure or the loading is asymmetric. This difficulty in predicting the correct failure mechanism precludes the mechanism method's general use.

Membrane analyses (Heyman, 1977) may not give the actual forces in a masonry vault. The position of the actual surface of thrust in the vault is unlikely to coincide with the middle surface of the vault, which is usually used in a membrane analysis. Membrane analyses are suitable for masonry vaults which are within discontinuities and whose loads are continuous, i.e. no cracks and/or point loads. A membrane analysis generates an acceptable force system and therefore a safe lower bound on the collapse load

None of the methods for masonry shell analysis mentioned is general. Mechanism analyses are unsuitable unless the collapse mechanism is known with certainty. Membrane analyses have difficulty with discontinuities in the structure or load pattern and Heyman's slicing technique has difficulties representing structures which span in two directions simultaneously.

3.6 Force network model

The line of thrust, which represents the state of stress in a linear arch, can be drawn as a series of straight lines. If the loads upon the arch are discretized into a set of point loads then the line of thrust is found by constructing a funicular polygon. In the same manner the surface of thrust, which describes the state of stress in masonry vaults (Heyman, 1977), can be represented by a network of forces. The nodes of such a network are points on the surface of thrust and the forces within the vault are represented by the network of forces which interconnect the nodes.

Modeling the surface of thrust as a system of discrete forces facilitates the analysis of vaults subjected to discrete loads. The distributed loads on the vault, such as self-weight, can be modeled as a set of discrete point loads. The accuracy of the representation is dictated by the network's mesh density.

The forces in a masonry vault must be in equilibrium with the loads applied to the vault. If the masonry is assumed to have no strength in tension then the surface of thrust must lie within the masonry and all the forces in the masonry must be compressive.

Therefore, the forces in the network model cannot be tensile, the forces meeting at each node in the network model must be in equilibrium with the loads applied at that node and the network must lie within the masonry envelope.

The application and optimization of force network models in the analysis of masonry vaults is described in the following steps. The steps outlined below describe an analysis to find the collapse load factor for a given pattern of imposed loads.

The identification of the load path is the starting point when trying to discretize a vault into arches. The load path will correspond to compressive forces, since arches are subjected mainly to compressive forces and Heyman's condition established that no tensile forces act in the arch when using limit analysis,

In this particular case a FEM models using SAP2000 will be used to better understand the structural behavior by plotting the minimum principal stresses as vectors. Pathologies, loading criteria and boundary conditions should be also taken into account, since the results are nightly sensitive to small variations in these factors.

Once the networks of arches are defined, the applied loads are also discretized depending in the loading criteria. Vertical and horizontal components transferred by one arch to another are vital to guarantee equilibrium.

The live loads are applied producing the worst case scenario for the vault. Since the compressive strength of the vault is usually high compared to the acting forces, is preferable to apply the loads producing the instability of the vault rather than increasing the magnitude of the acting forces. A live load distribution comparison is discussed in Appendix B.

If a safety state is encountered (A thrust line inside the network of arches) the computation of the safety factor is the next step. An iterative procedure will maximize the value of Lambda that each arch is capable of sustain, the safety factor is represented as a live load amplifier. The steps necessary to maximize Lambda are summarized in *Figure 56*

In order to determine the overall safety of the vault, the minimum Lambda factor encountered in the arches will represent the overall safety factor of the vault. Since different models are proposed when proposing network forces capable of representing the structural behavior, by the judge of the engineer and making a reasonable comparison Lambda can be determined. The general procedure to assess Catalan vaults is described in *Figure 11*.

This procedure will be follow in the two case studies to be presented in the following chapters, the assessment of the two curvature Catalan vaults will give us enough information to make a good judgment on the safety of these vaults and define if an intervention is needed.

The methodology is summarized in the following flow-chart in Figure 10. The aim of this methodology is to assess the safety of the vault under different conditions.

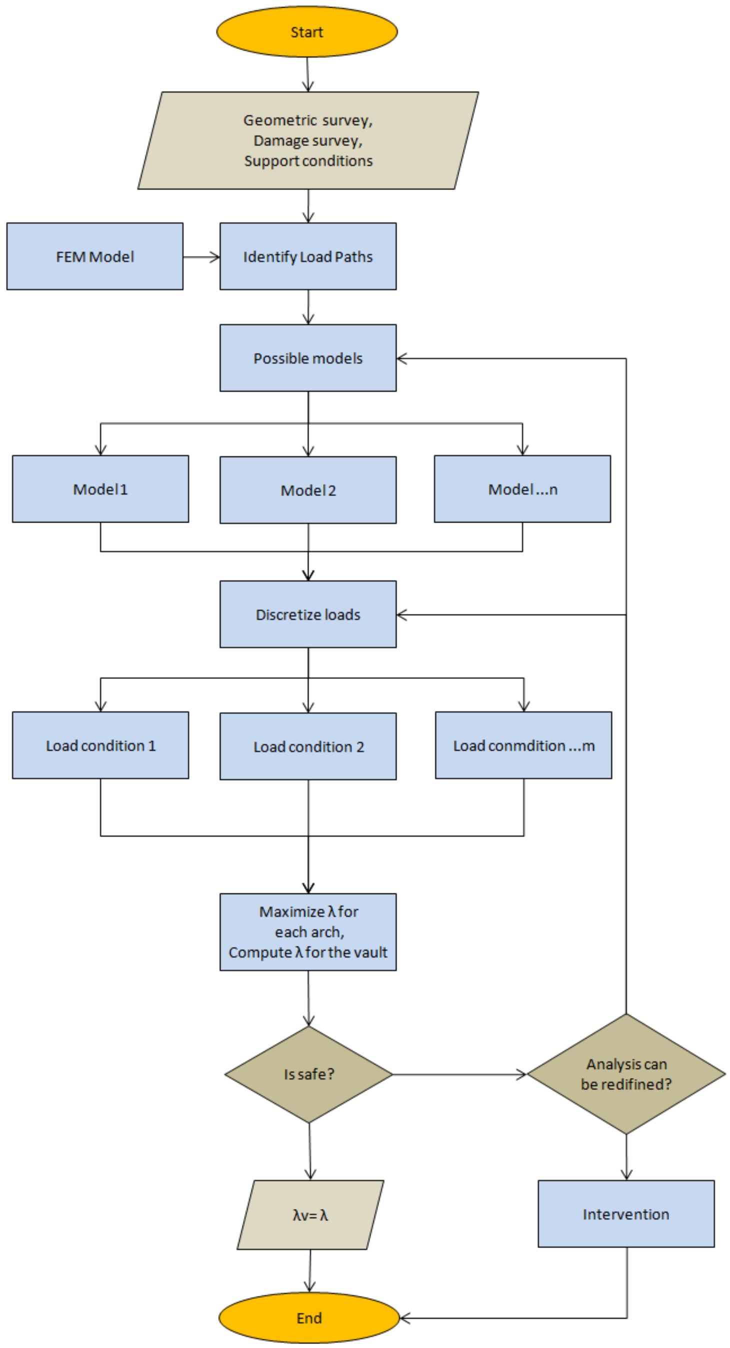


Figure 11: General procedure to assess Catalan vaults

4. FIST CASE STUDY (VIC CHURCH)

4.1 Introduction

The Temple of Saint Anthony M. Claret is located in Vic which is the capital of the region of Osona in Barcelona, is located on the banks of the Meder River, in the vicinity of the impressive Sau reservoir. The Temple lodges the remains of who in life was Anthony Mary Claret (1807 –1870) , a Catalan Spanish Roman Catholic archbishop and missionary, who was confessor of Isabella II of Spain and beatified in Rome by Pope Pius XI in, 1934.

Figure 12: External view and façade of Temple Saint A. Maria Claret

The Temple was built following a classic architectural style during (1957-1970) by the architect Josep Maria Rivas i Casas.

4.2 Structural components

The church is a clay masonry construction, it has a rectangular plan view (22 X 42) m, and it presents three naves in the longitudinal direction. A system of arches spanning in the short direction supports a series of double curvature Catalan vaults. The reinforced concrete arches are 77 cm wide and 60 cm depth and are spanning 4.5 meters center to center. See *Figure 13*

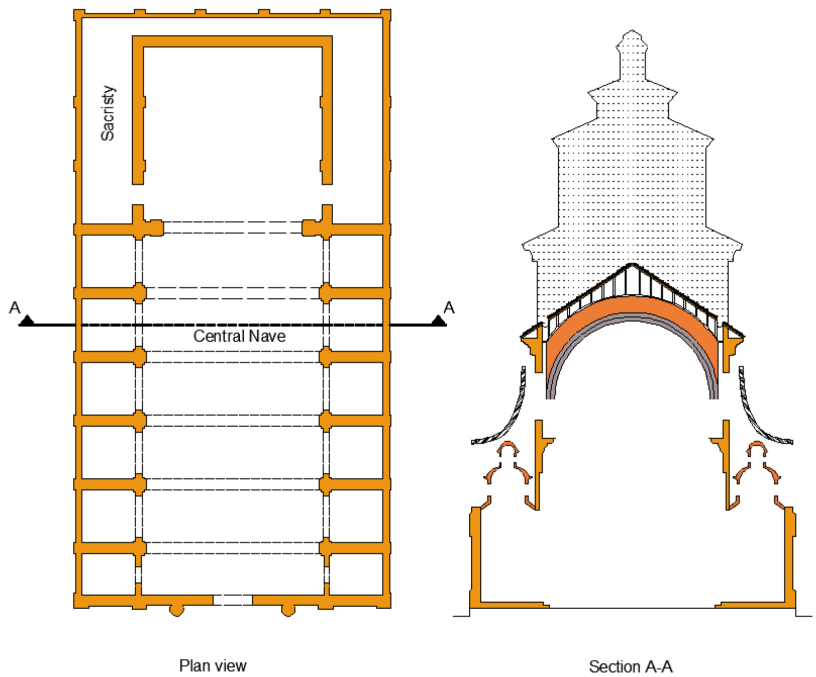


Figure 13: Plan view and section of the Temple

The arches are supported by buttresses 5 m wide and 17 m tall and they transfer the horizontal thrust from the arches to the foundation. See *Figure 14* and *Figure 15*

Figure 14: Buttresses supporting the Temple

Figure 15: Construction procedure of the Catalan vaults

The Catalan vaults consist of three layers of terracotta tiles interlocked with layers of mortar and have an overall thickness of 10 cm. The vaults support a series of lightweight partition walls which transfer the load from the upper roof.

The six vaults in the central nave present the same geometry and as it will be seen after similar crack patterns. The span in the transversal direction of the nave is 12 m and in the longitudinal direction is 4.5 m.

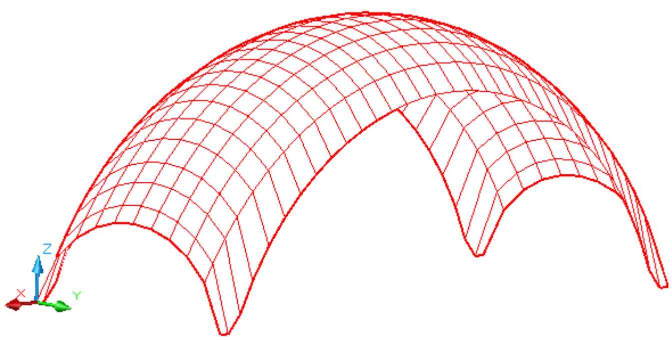


Figure 16: Geometry of the Catalan vault

4.3 Pathologies and damage identification

Looking at typical pathologies can help to better understand how the structure might be acting. For example no load path can travel perpendicular to the open cracks since there is not contact point to transfer the compressive forces.

Cracks can also be due to a deficiency in the construction procedure. For example the action of the horizontal thrust in the lateral vaults in which the forces are not counterbalanced can produced detachment of the vault.

The existing pathologies have a great impact on the analysis, which results are highly sensitive to minor changes in the loading or support conditions. Different hypothesis should be considered based on the damage identification.

During the site visit of the Temple several crack patterns in the vaults were found. These cracks were mapped in the following picture and it can be seen that are located at the springing and mid-span of the vault in the short direction. The vaults seem to be detached of the arches.

These cracks corresponds to de the damage found in the intrados only, and this damage will be taken into account during the analysis for better understanding the structural behavior or correlate some results from the analysis.

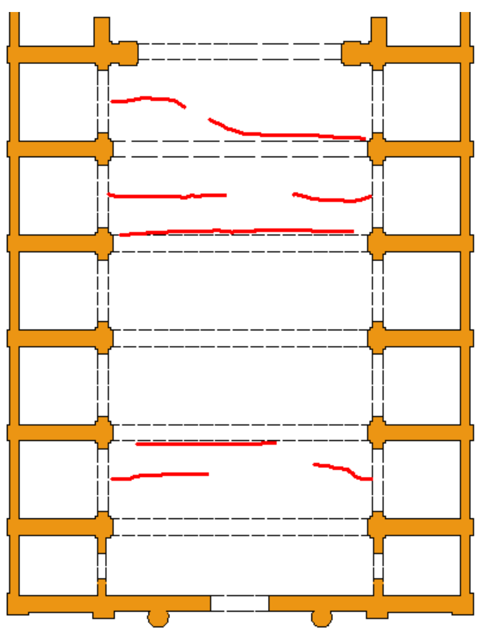


Figure 17: Crack mapping in the vaults of the central nave

Figure 18: Crack in the longitudinal direction of the vault

This detachment of the vaults out of the arches can be explained by the hypothesis of a high horizontal thrust on the extreme vaults which cannot be cancelled by other horizontal resultant as the middle vaults. The other hypothesis suggests the formation of the hinges at this location due to a hinging mechanism in the vault.

4.4 Structural Analysis

The structural analysis as described before will be focused on the Catalan vaults, different load path distribution and supporting condition will be taken into account based on the geometry, damage maps and the help of a Finite Element model.

The different models to be analyzed using the limit analysis approach, have the aim of simplify the analysis by decomposing the vault in a series of arches as it was explain in the previous chapter.

The spreadsheet used for the analysis uses the equilibrium equations to build up graphically the thrust of line and assess the safety of an arch. The assumptions and formulations behind the spreadsheet and further information of how graphic static is applied, is discussed in Appendix A.

4.5 Loading criteria

The self-weight of the upper roof is transferred to the vault trough the supporting walls, the live loads in the same way. Two different live load conditions were considered for the analysis, the first one considering a snow load uniformly distributed with a magnitude of 1kN/m2. The second load condition is the maintenance load which has the same magnitude of the snow load but it can be applied in different ways.

From the stability point of view not the magnitude but the position of the load will be more critical at the moment of assess the vault. In principle the snow load will produce higher stresses on the vault, but since the design is governed by the stability and not the strength of the material, the maintenance load can be more critical.

For the FE model the loads were applied where the walls are located, for this reason the mesh of the vault matches the position of the supporting walls, and the live load and roof self-weight were applied as punctual loads in the nodes of the model. See Figure 19

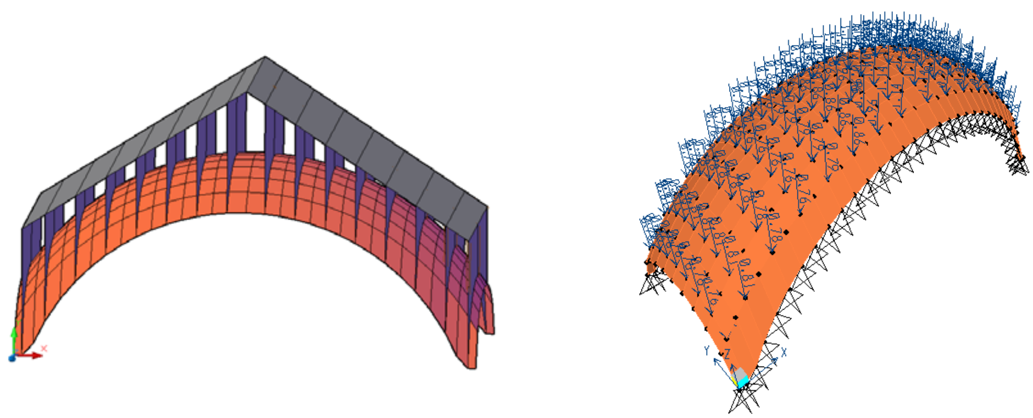


Figure 19: Roof supporting system and FE model loading criteria

4.6 Simplified models

The main difficulty of this analysis procedure, is try to understand how the loads travel trough the vault to the supports and the load paths describing the principal compressive forces based on the loading condition and support conditions.

In order to better understand the “actual” state or behavior of the vault a simplified FE model was created in SAP 2000. By performing a simple static analysis of the vault and displaying the minimum principal stresses is easier to predict or better estimate the way the compressive forces travel along the vault.

If necessary changes in the boundary conditions and the loading criteria can easily implemented in the FE model, and multiple scenarios can be assessed.

Model 1-a

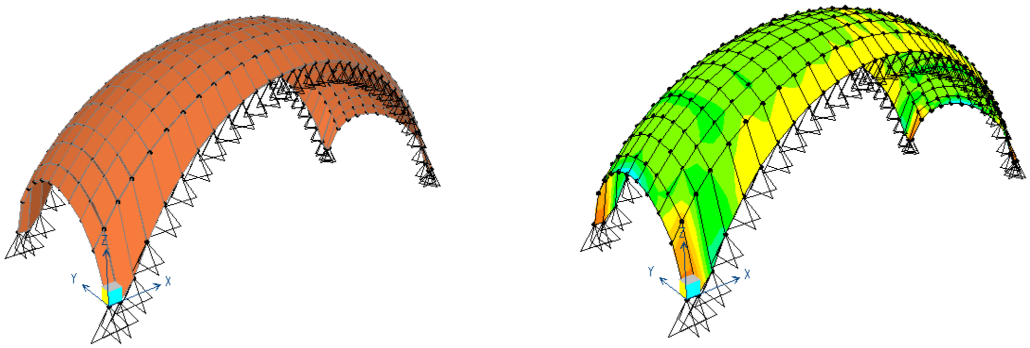


Figure 20: Finite element model and minimum principal stresses contour-maps

The stresses distributions are highly sensitive to minor changes in the vault boundary conditions. For this reason multiple scenarios should be considered. The first hypothesis is based on the visual inspection of the church. As shown in *Figure 17,* it can be seen that the hinges are located perpendicular to the short dimension of the vault in a regular basis, this pathology give us an insight of a unidirectional behavior.

Based on this hypothesis the first model was defined and contour-maps were used to verify the unidirectional behavior.

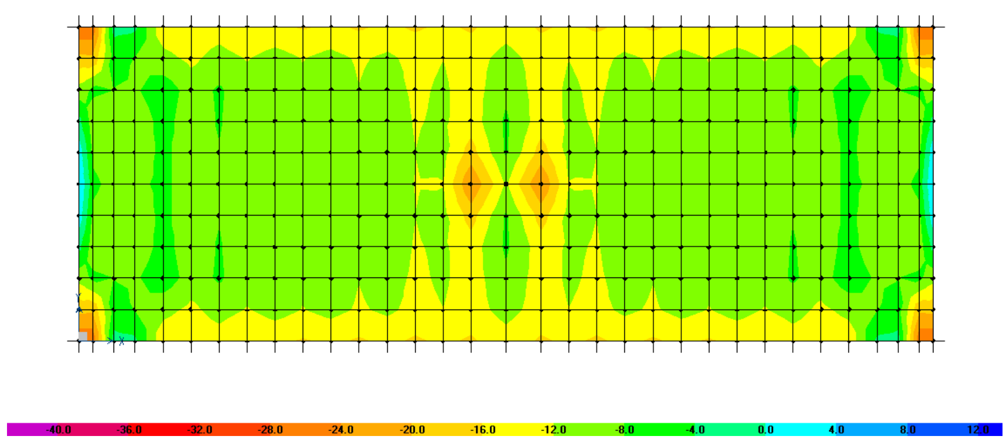


Figure 21: Contour-maps for principal minimum stresses Model 1-a

A simple hypothesis which consists in dirscretizing the vault in a series of parallel arches was defined and the limit analysis approach was used using the spreadsheet developed for the analysis. See *Figure 22.*

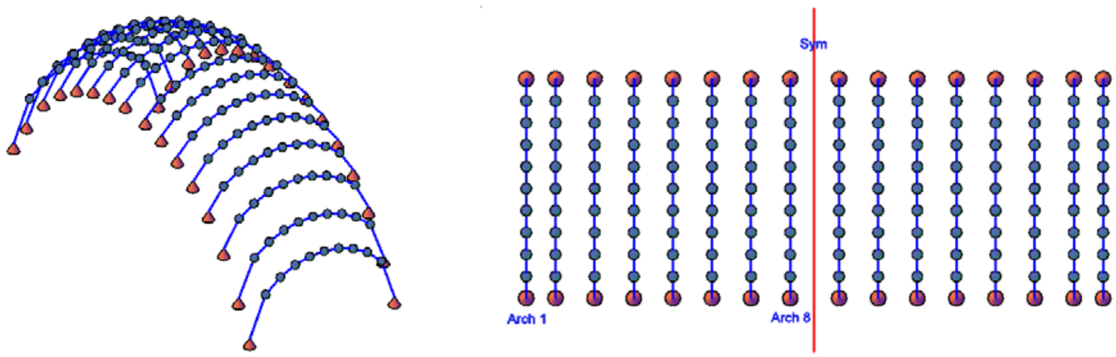


Figure 22: Vault discretized as a series of arches, Model 1-a

The parabolic arches with a fix span, but different rises were evaluated to withstand their own weight, superimposed loads and live loads. If no thrust limits are imposed and the applied live load remains uniformly distributed, the arch will never collapse, since there will be always a horizontal thrust capable of withstand the live load increment.

Is for this reason that an evaluation on the buttresses stability has to be performed and also a study in possible conditions in which live loads are applied. The critical scenario will govern the assessment of the vault.

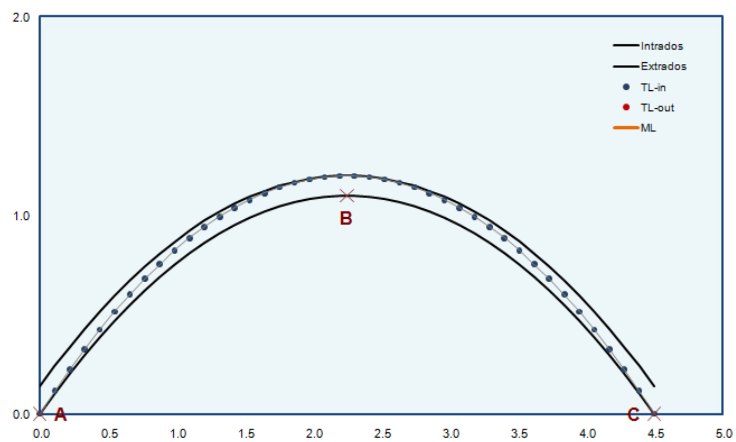


Figure 23: Thrust line corresponding to the arch 1, subjected to distributed live loading (Horizontal thrust of 3.3 KN and vertical reaction of 6.5 KN)

In order to maximize Lambda the uniform distributed load was amplified and the vertical reactions transferred to the supporting arch. See *Figure 24.* Once the vertical loads are transferred to the supporting reinforced concrete arch, the stability of the buttresses was evaluated.

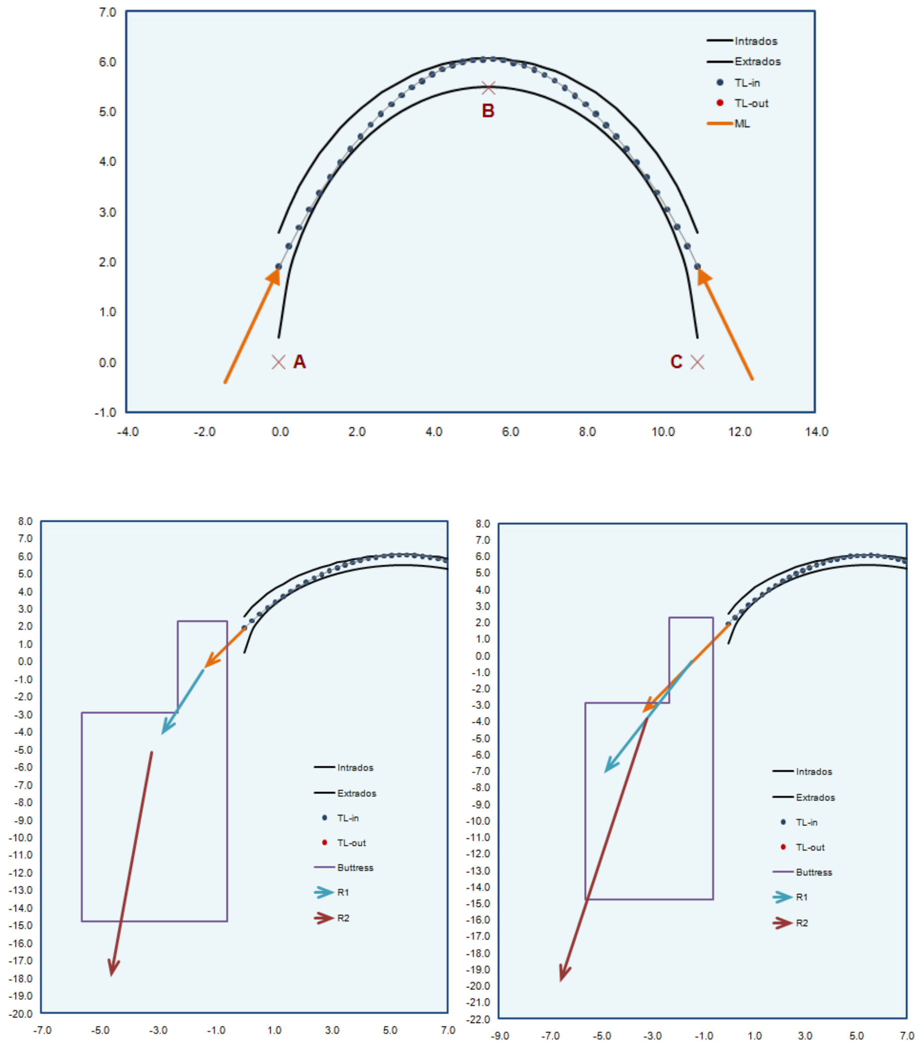


Figure 24: Buttress stability analysis

A safety factor of 10 has to be applied to the uniformed distributed live load in order to make the buttress unstable. Different live load patterns will be studied in order to minimize this factor by distributing the live load in a position that makes the structure more unstable. (*See Appendix B: Live load comparison*).

Model 1-b

In order to encounter the critical state from which the value of Lambda will be decreased, a second model assuming the application of live load only at one quarter of the span as punctual load was considered. In the live load distribution study, carried out in Appendix B it is shown that by applying a distributed load in half of the span will produce the most critical case, but since this situation is not realistic either for a maintenance load or snow load, the punctual load at one quarter of the span was used.

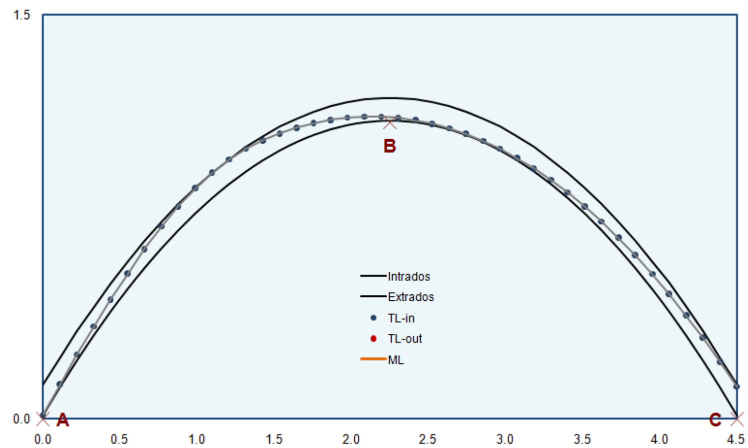


Figure 25: Thrust line corresponding to the Arch 1, subjected to a punctual live load applied at one quarter of the arch

Due to symmetry conditions eight arches were evaluated and the maximum Lambda values obtained from the arch analysis are shown in the next table:

Arch	Safety factor	Rise(m)
Arch 1	2.6	2.05
Arch 2	2.6	1.85
Arch 3	2.7	1.50
Arch 4	2.8	1.33
Arch 5	3.1	1.22
Arch 6	3.3	1.16
Arch 7	3.6	1.12
Arch 8	4.0	1.10

Table 1: Safety factors for Model 1-b

The overall safety factor of the vault would be given by the lowest of these values (Arch 1 – SF. 2.6). The procedure of maximizing Lambda is shown in *Figure 36.*

Model 2-a

In the previous model, the hinge observed in the mid-span in *Figure 23* matches the crack presented in the crack mapping in *Figure 17*. But it is important to mention that this mechanism is not consistent in the springing, since the cracks should be placed in the extrados and not in the intrados.

This apparent detachment of the vault from the arch may be attributed to the horizontal thrust taken by the extreme arches of the nave, or a construction imperfection. In any case this should be taken into account in the analysis and the supports removed from the edges where the detachment exists.

This worst case scenario was also modeled in SAP200 in order to better understand the way in which the compressive forces travel through the vault and try to establish a model capable of match this load path. The support where removed where the vault is detached from the supporting arch.

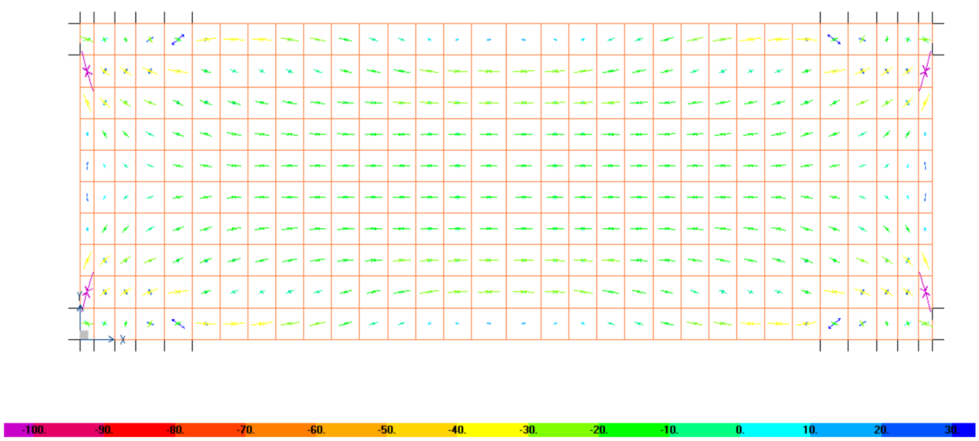


Figure 26: Contour-maps for principal minimum stresses Model 2

A first model was build taken into consideration the minimum principal stresses contour-maps showed in the previous figure, this poly-funicular model tries to simulate how the weight of the central vault can be represented as an arch and transfer its weight to two supporting arches which transfer the load to the corners of the vault due to the absence of lateral supports. See Figure 27

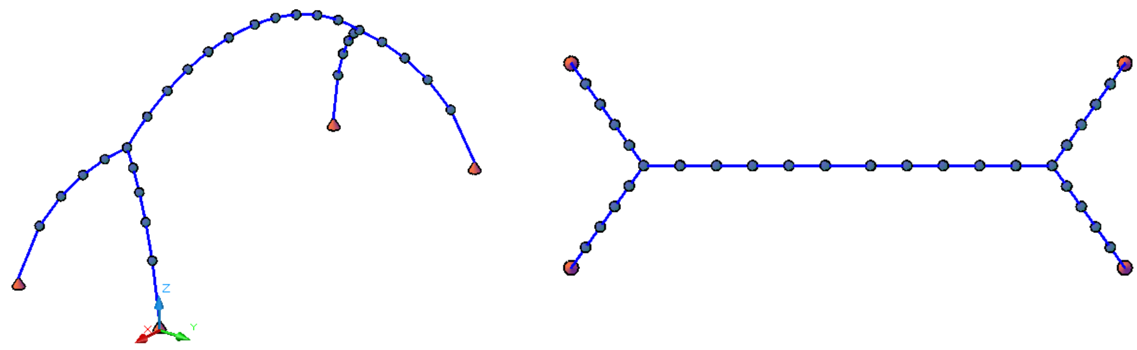


Figure 27: Poly-funicular of Model 2

This first attempt to make a model able to simulate the actual behavior of the vault showed to be inadequate, since the vertical load transferred by the central arch is too high and the supporting arches are not capable to sustain this load as seen in *Figure 28*.

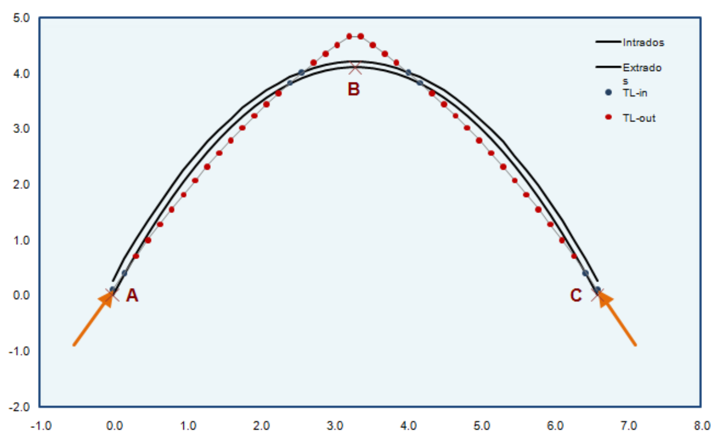


Figure 28: Thrust line of supporting arch – Model 2-a

A refinement on the model is needed as suggested in the methodology showed in *Figure 11*, this first attempt suggested the alternative of better redistributing the loads in a more uniform way. Model 2-b-1 will consider 5 arches instead of just one, distributing the vault central weight to the other two supporting arches.

In this model the action of horizontal forces will be disregarded to simplify the analysis. Model 2-b-2 will consider the action of horizontal forces.

Model 2-b-1

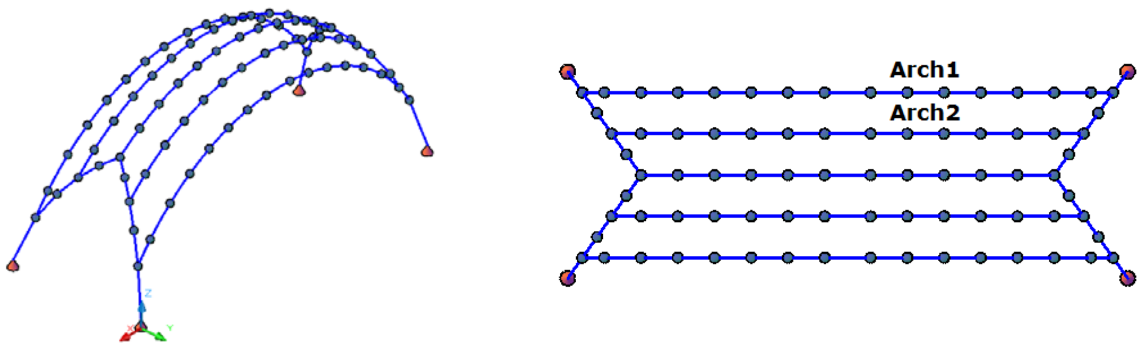


Figure 29: Poly-funicular of Model 2-b

A thrust line inside the supporting arch was found; nevertheless this procedure is too conservative since horizontal actions were disregarded. See *Figure 30*

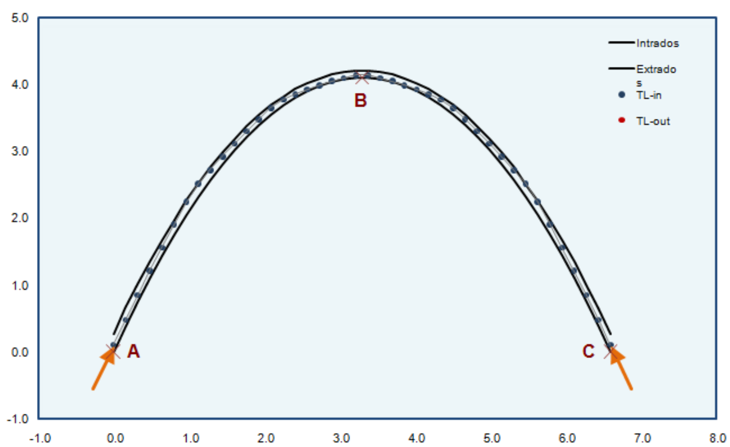


Figure 30: Thrust line of supporting arch Model 2-b

The thrust line showed in the previous figure corresponds to a uniform distributed load. If the live load act as distributed, the vaults will fail under high Lambda coefficients and the stability of the buttresses are to be verified as seen in Model 1.

It is for this reason that a punctual live load will be applied where it produces the maximum instability to the system of arches. For example by applying a punctual load in one of the lateral central arches will produce an asymmetric load condition which will minimize Lambda.

Arch 1 was subjected to a punctual live load at a quarter of the span. By increasing this value by a factor of 2, the supporting arch thrust line went out of the thickness of the arch. This punctual load represents the maintenance load and from the stability point of view it is better to apply it to one arch.

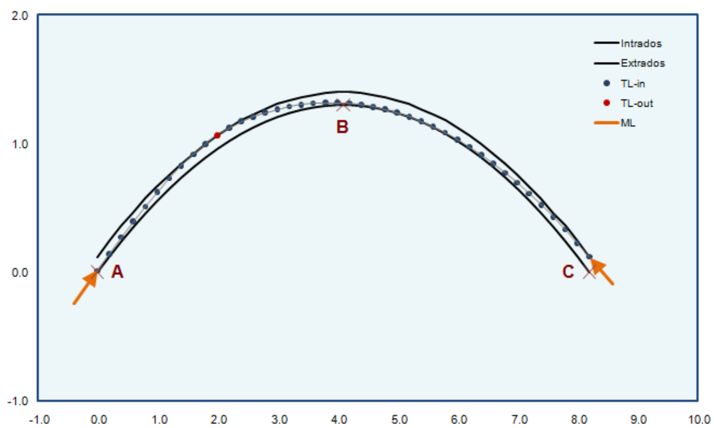


Figure 31: Unsafe condition in supporting arch, reached by using a safety factor of 2, in arch 1

Likewise arch 2 was analyzed and a similar safety factor was encountered. These values of Lambda are low, but considering that only the supports at the corners are considered is seems reasonable to get low safety factors.

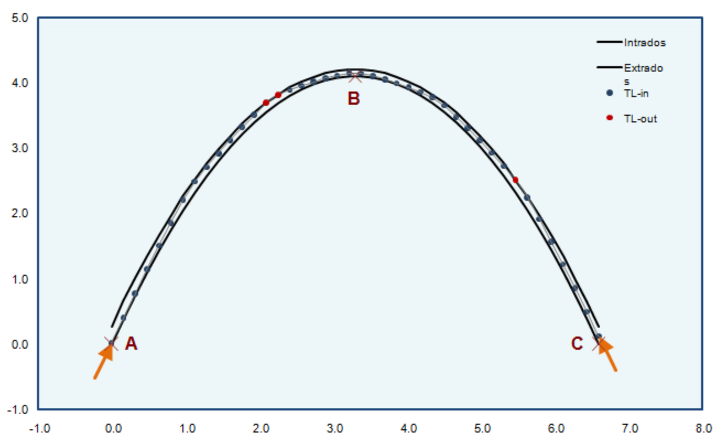


Figure 32: Unsafe condition in supporting arch, reached by using a safety factor of 1.9 in arch 2

Then the overall safety factor of the vault will be 1.9 which correspond to the lowest value from the analysis.

Model 2-b-2

This model takes into account the action of the horizontal forces coming from the central arches. Horizontal and vertical thrust components transferred from the upper arches in the force network were combined with the components from the supporting arches in order to get the real structural behavior of the arches under compressive forces.

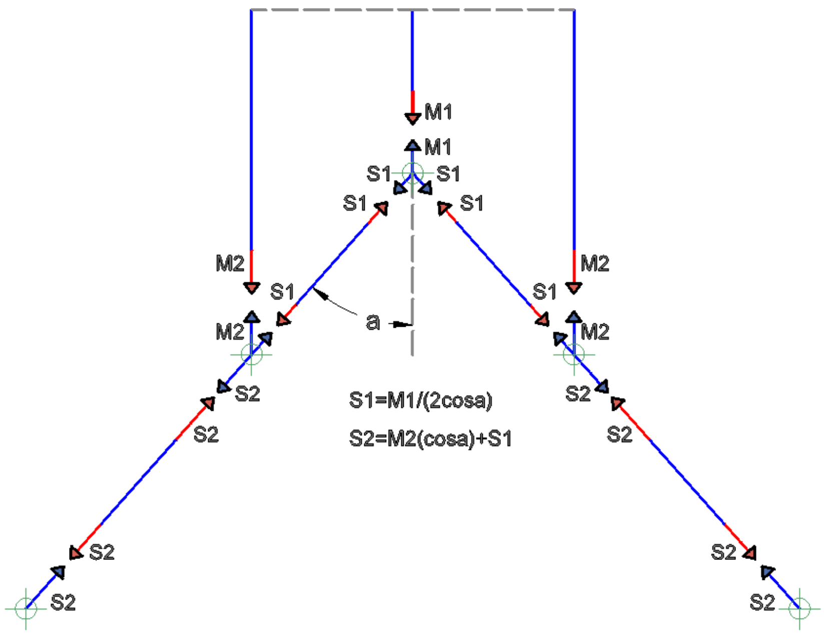


Figure 33: Vector decomposition of forces coming from the central arches

A safe condition was found by applying a horizontal thrust of 36.8 KN in the supports. The analysis was carried out under uniform distributed live load. The supporting arches were divided in segments to better understand how the forces merge and travel towards the supports. See *Figure 34*.

In order to simplify the analysis a uniform distributed live load was used for the analysis. As seen in *Figure 33* the horizontal component coming from the central arch merge with the horizontal components from the rest of arches. Last model did not take into consideration this compressive increment in the arch. These equations are obtained after applying the equilibrium equations in the nodes where the forces were merging.

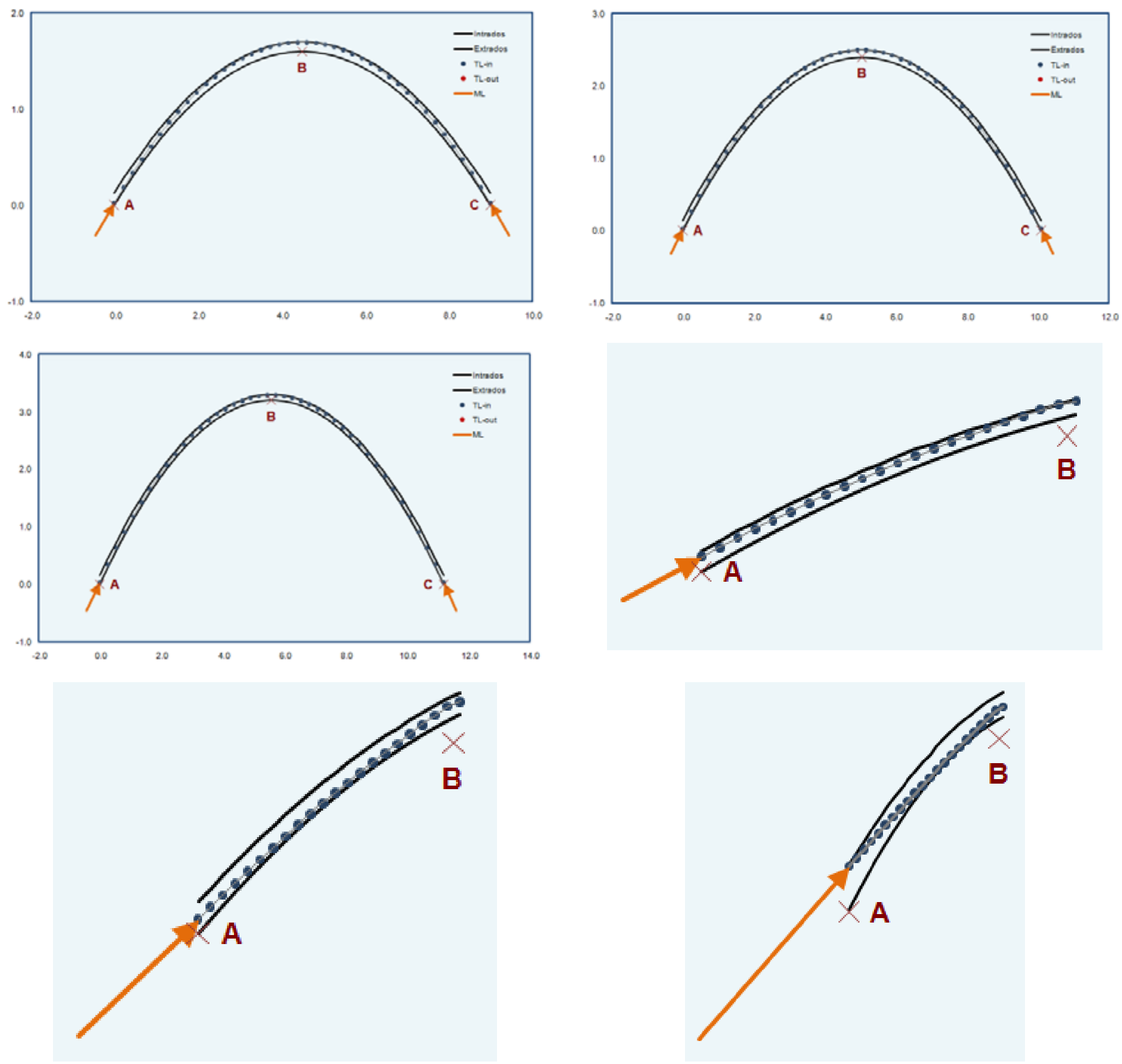
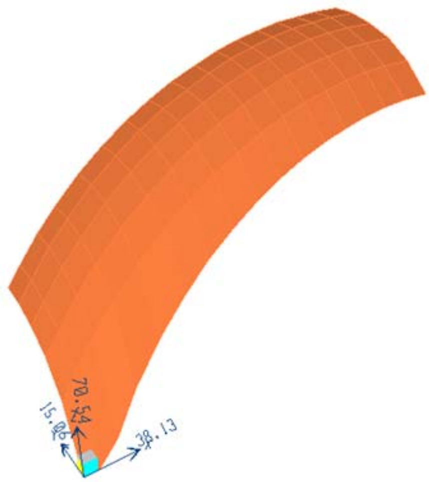


Figure 34: Thrust line inside supporting arch, horizontal thrust components were taken into consideration

The spreadsheet as explain in Appendix A, by changing the horizontal thrust and the thrust line location at the springing, will build up the thrust line. The safe condition was encountered since the thrust line lies inside the arch thickness. The horizontal applied force transferred from the arches to the supporting arch was applied at the right top end. These horizontal forces were obtained after applying the formulae shown in *Figure 33.*



OutputCase Text	CaseType Text	GlobalFZ KN
DEAD	LinStatic	105.03
LIVE	LinStatic	53.64
SDEAD	LinStatic	123.48
U1	Combination	282.15

SV	SH
-72.7	36.8

Figure 35: Comparison of corner reactions with FE Model

The summation of the vertical and horizontal components obtained from the poly-funicular analysis for Model2-b-2 was compared with the total weight of the vault obtained from the SAP2000 Model. In the vertical direction a difference of 3% was encountered and in the horizontal direction 5%. This small difference guarantees good force redistribution within the arches.

4.7 Assessment on the safety of the vaults

In model 1 and Model 2 a safe condition was encountered, Lambda showed to vary from 1.9 to 10 depending in the way live loads are applied and the arch geometry. An extreme case scenario was considered by removing the lateral supports in both sides of the vault due to the detachment of the vault from the arch and still a safe condition could be demonstrated.

The elastic analysis performed using SAP2000 was useful since the vertical and horizontal reactions were compared with the different models to check the ability of the model to correctly transfer the forces throughout the vault to the supports.

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5. SECOND CASE STUDY (PALAU LIO)

5.1 Introduction

The museum is located in Montcada Street in the Gothic quarter of Barcelona. The Montcada Street in the Borne Area is full with all kind of palaces and small palaces, homes of wealthy nobles and merchants from the medieval period (1350). Many of these Palaces are used now as Museums, Art Galleries or Bars.

The history of the Museu Tèxtil i d'Indumentària dates back to 1883, when Barcelona City Council acquired its first collections of textiles with the aim of creating a museum dedicated to the subject. For a good part of the 20th century, the collections of fabrics, clothing and lacework were split between various museums. In 1964, the Museu Tèxtil opened in the Palau del Marquès de Llió thanks to a donation by Manuel Rocamora.

Figure 36: Interior and exterior view of the Textile Museum

5.2 Structural components

The structural components that are related to the vaults to be analyzed are in a room located in the basement of the Palace. A circular column is located in the middle of the room supporting in cooperation with the masonry walls four Catalan double curvature vaults.

The walls are made of stone masonry and are approximately one meter thick, these walls since are in the basement serve as a support for the upper walls (2 floors).

The four arches are approximately 40 cm wide and act as supports for the vaults where there is absence of walls.

5.3 Pathologies and damage identification

During the site visit a damage mapping on the vaults was performed, two main cracks were found, all of them in the same vault and its location is indicated in the next Figure.

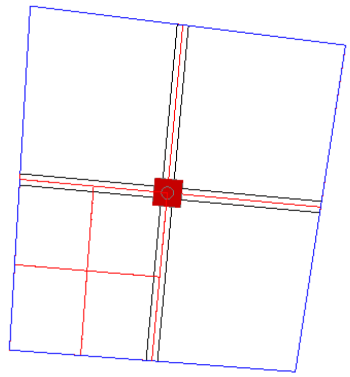

Figure 37: Position of the vault to be analyzed

These cracks are mapped in the next Figure and two of them are located in the mid-span in the extremes of the vault. It is also a noticeable material replacement in the central part of the vault.

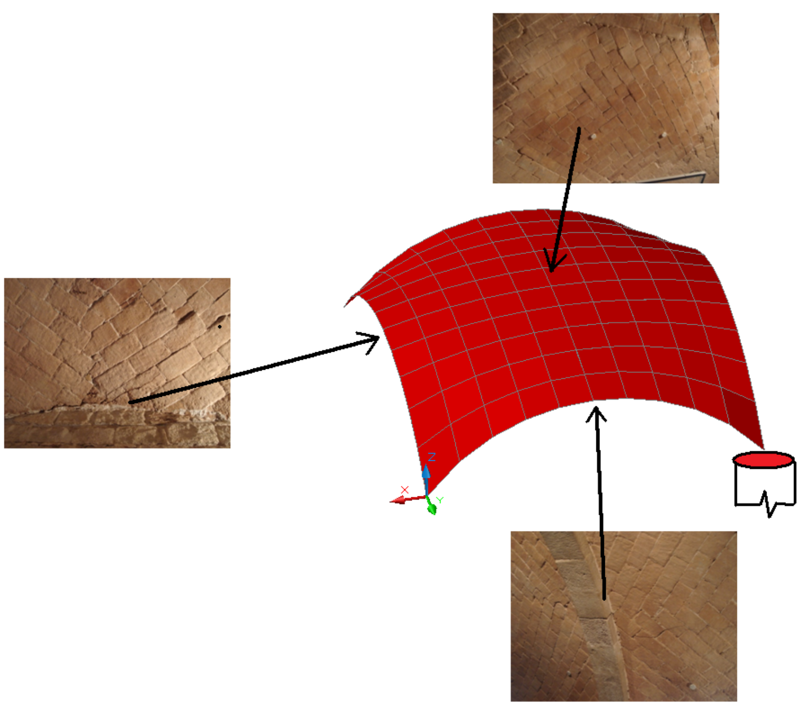

Figure 38: Geometry of the vault and damage survey

5.4 Loading criteria

In order to better estimate the permanent loads acting over the vault, the information available from a drilling study in the vault was used.


Figure 39: Pit location and internal view

Similar to the previous case study a system of masonry supporting walls was found in the upper part of the vault. This supporting system transfer the loads coming from the upper slab. A self weight of 1KN/m2 was considered for the slab and the weigth of the supporting walls was uniformly distributed (1.9 KN/m2).

The live load to be considered was taken from the NBE-AE-88 *Acciones en la Edificacion* and corresponds to a comerciall use live load of 2kN/m2.

5.5 Structural Analysis

Since the geometry of the four vaults encountered in the basement have similar geometry and apparently there are not problems related to the instability of the supporting elements (Due to the cancelation of the horizontal thrust by symmetry in the column and the thickness of the walls and the upper walls weight contribution), the simple analysis of one vault isolated is justified.

The model to be analyzed using the limit analysis approach, have the aim of simplify the analysis by decomposing the vault in a series of arches as it was explain in the previous chapter.

The spreadsheet used for the analysis uses the equilibrium equations to build graphically up the thrust line and assess the safety of an arch. The assumptions and formulations behind the spreadsheet and further information of how graphic static is applied, is discussed in Appendix A: Arch analysis optimization

5.6 Simplified models

The main difficulty of this analysis procedure, is try to understand how the loads travel trough the vault to the supports and the load path of the compressive forces, based on loading condition and support conditions.

In order to better understand the "actual" state or behavior of the vault a simplified FE model was created in SAP 2000. By performing a simple static analysis of the vault and displaying the minimum principal stresses is easier to predict or better estimate the way the compressive forces travel along the vault.

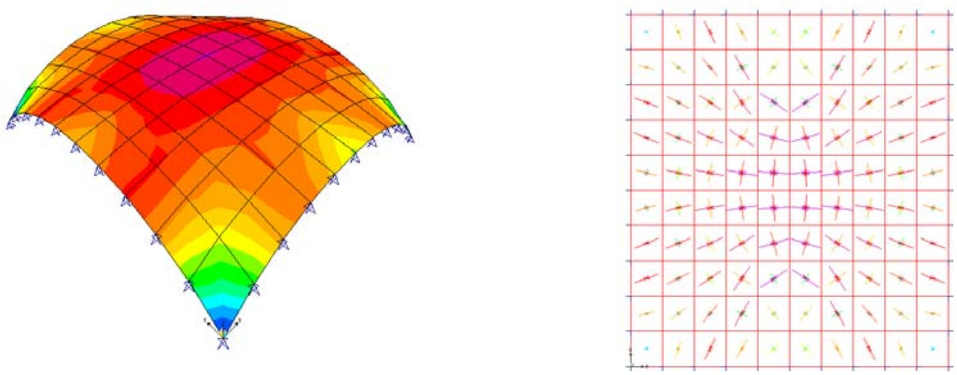

Figure 40: Contour-maps for principal minimum stresses

Model 1-a

After the FE Model was carried out and the principal minimum forces displayed, a first model was proposed. This model does not take into account the horizontal action since the horizontal forces act almost horizontally and can be considered in equilibrium.

A uniform distributed live load will be used for the analysis. Since the arches to be analyzed are symmetrical due to the network shape and vault almost-square form, the analyses of only three arches are to be performed.

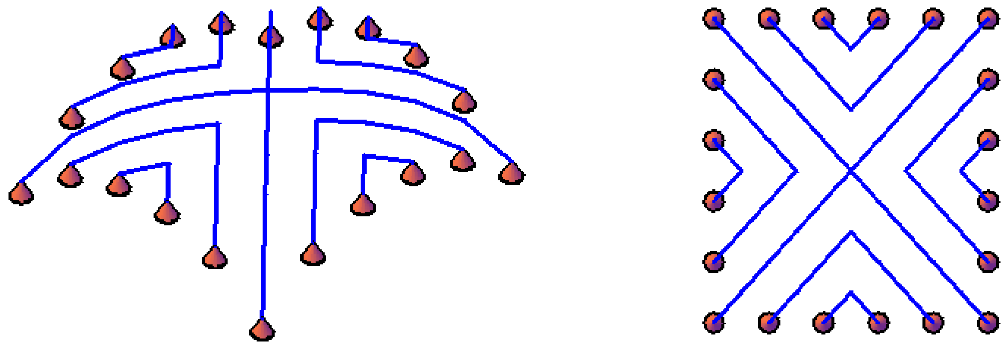

Figure 41: Vault discretized as a series of arches, Model 1-a

The loads were discretized considering their respective tributary areas corresponding to each of the arches to be analyzed (See *Figure 42*). The loads where applied in the arches as punctual loads on each voussoir that was considered for the analysis.

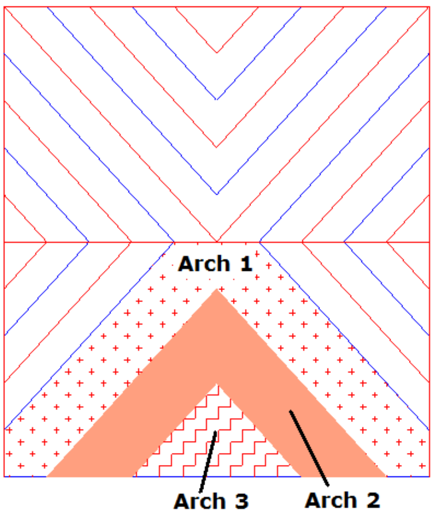

Figure 42: Tributary areas, Model 1-a

The analysis was carried out and a safe condition was reached for each of the arches. Arch 1 minimum thrust line was found when applying a horizontal thrust of 16.5 KN; Arch 2 thrust line was encountered with a horizontal thrust of 13.4 KN and arch 3 required just 3.4 KN.

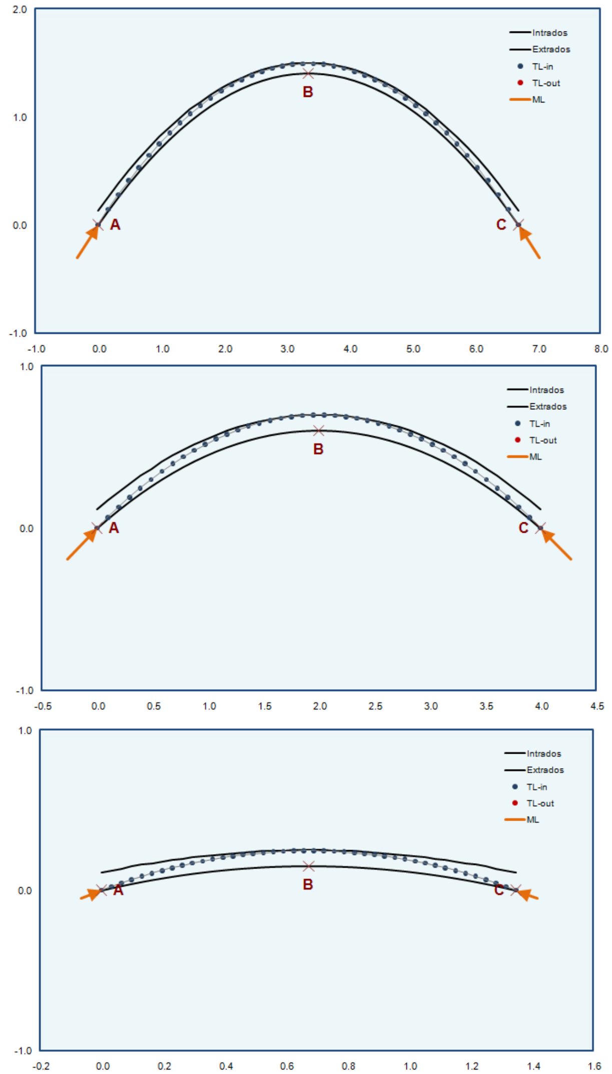

Figure 43: Minimum Thrust line encountered for each set of arches.

Vertical equilibrium was double checked by comparing the summation of the vertical reactions on the FE Model. The summation in the spreadsheet gives 155 KN in the vertical direction which is close enough to the value 153.8 showed in the figure below.

	OutputCase Text	CaseType Text	GlobalFZ KN
	DEAD	LinStatic	43.598
	SDEAD	LinStatic	65.246
	LIVE	LinStatic	44.955
▶	U1	Combination	153.799

Figure 44: FEM Reactions summation on the Z direction

Model 1-b

Horizontal forces were disregarded in the previous model since the symmetry of the vault and the way live loads are applied are favorable to the model and this horizontal action is considered to be in equilibrium.

Although if asymmetric loading is considered, the horizontal action will not be in equilibrium and model 1-a. would fail representing this case. Therefore small changes in the forces network are implemented in Model 1-b. See *Figure 45*

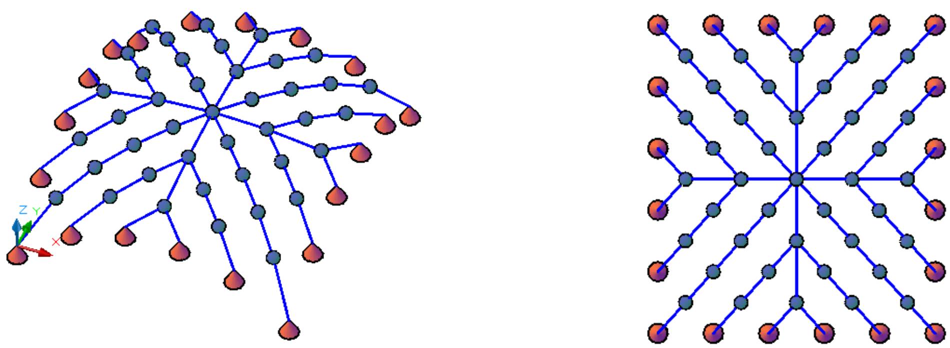

Figure 45: Poly-funicular of Model 2

In order to maximize Lambda, the worst case scenario was considered. This considers in applying the live load in one half of the vault, this loading scenario will increment the difference between the horizontal forces coming from the arches. The safety factor is to be amplified until the arches on the opposite side of the loading are incapable of withstand the horizontal load increments.

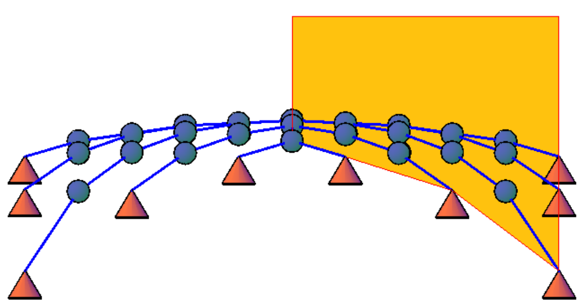

Figure 46: Live load distribution, Model 1-b

In order to quantify the magnitude of unbalanced horizontal force, the arches were evaluated and the forces added in the merging points. The asymmetry of the load will produce an unbalanced horizontal load which has to be redistributed among the arches.

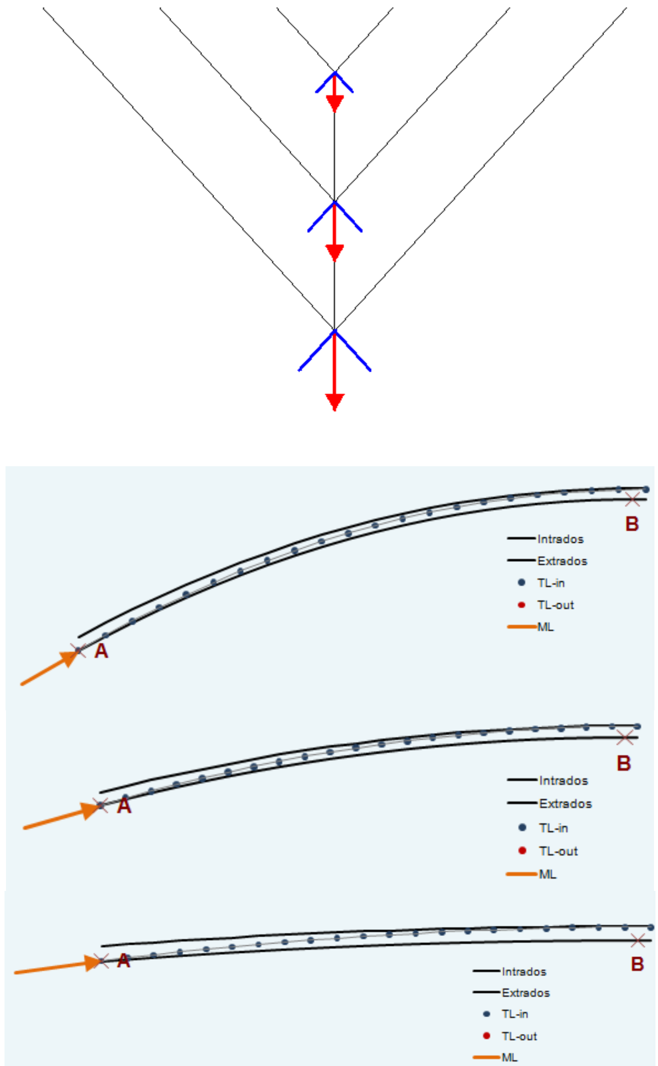

Figure 47: Analysis of the arches under asymmetric loading

Once the horizontal forces were calculated and the horizontal unbalanced force calculated (See Figure 48), the arches were analyzed in their capacity to withstand the additional forces. This procedure was carried out until Lambda produced a set of forces impossible to withstand for the arches.

Uniform distributed Load

	H	V	Hh
Arch 1	17.3	15.27	24.5
Arch 2	14.1	9.66	19.9
Arch 3	3.6	2.55	5.1
			49.5

Non-Uniform Load

	H	V	Hh
Arch 1b	12.5	11.07	17.7
Arch 2b	10.1	6.93	14.3
Arch 3b	2.4	1.71	3.4
			35.4

Hincr	14.1

Arch 1c	50%	7.1
	25%	3.5
Arch 2c	50%	3.5
	25%	1.8
Arch 3c	50%	1.8
	50%	1.8

Figure 48: A Lambda coefficient of 2 produced a horizontal increment of 14.1 KN

In order to redistribute the horizontal increment a simple FE model was used to have an idea on the forces redistribution.

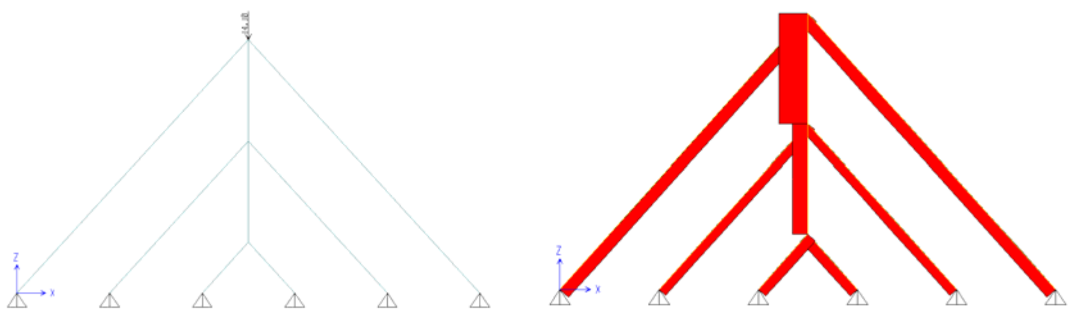

Figure 49: Horizontal increment redistribution among the arches

The force of 14.1 KN was redistributed among the arches and arch 1 was found incapable of withstand the horizontal increment, therefore Lambda was established as 2.

5.7 Assessment on the safety of the vaults

In model 1-a and Model 1-b a safe condition was encountered, Lambda was evaluated only under one live load condition which was believed to be the most critical. The safety factor of 2 is found to be low, but since the live load is high (due to the use of the building) and the live load was applied in asymmetrically this value seems to be reasonable and demonstrate the safety of the vault.

The elastic analysis performed using SAP2000 was useful since the vertical and horizontal reactions were compared with the different models to check the ability of the model to correctly transfer the forces throughout the vault to the supports.

6. CONCLUSIONS

The methodology presented for the study of three-dimensional masonry structures is based on the well known analogy between the equilibrium of masonry arches and that of hanging funicular systems.

In the search of a model capable of represent the behavior of the Catalan vaults, the elastic analysis performed in SAP2000 were useful to determine the load path and have a starting point in the process of making a model accurate enough to make a realistic assessment on the vaults under study.

The crack mapping is an important step previous the analysis, since it allows us to better understand the behavior of the structure by observing the different pathologies. These observations were included in the analysis either by changing the support conditions or the load conditions.

The methodology includes the development of spreadsheet with algorithms in the capacity of optimizing the analysis and to apply either the safety or lower bound or the uniqueness theorem of plasticity. The methodology can be used to assess the safety of masonry structures and to evaluate their ultimate capacity.

The comparison of the different models with the results from the elastic analysis has shown the accuracy of the method and its ability to deal with complex systems. Nevertheless further study is needed in the way the safety factor is computed.

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APPENDICES

APPENDIX A: ARCH ANALYSIS OPTIMIZATION

Introduction to the spreadsheet

As showed in previous chapters, a vault can be analyzed by discretizing it as a series of arches. In other words, this simplification converts a 3-D problem into a 2-D problem. This procedure implies another issue, which is the amount of arches to be analyzed; the necessity of developing a spreadsheet capable of running multiple arches with different geometry and different loading conditions is needed.

Geometry of the arch

In order to generate the geometry of the arch, the geometry parameters in conjunction with the type of geometry of the arch, either circumference of parabola has to be defined. The parameters showed in *Figure 50* will be used to construct the geometry of the arch (Intrados and extrados), compute the self weight and establish the limits for the thrust line (constrains).

Voussoirs	41
Span	6.7
Width	0.7
Height A	0
Height B	1.4
Height C	0
Unit weight	18

Depth A	0.1
Depth B	0.1
Depth C	0.1

Arch curve	P

Figure 50: Geometry parameters

Statically Indeterminate problem

In order to build up the thrust line, their springing reactions need to be calculated in the first place. For this purpose the equations of equilibrium can be used, but since the number of unknowns are six (Two reaction components in the X-Direction, two in the Y-Direction and the passing point of the thrust line at both ends) and the equilibrium equations are just three, one of this reaction components has to be assumed and the position of the springing as well, in order to solve the system indeterminacy.

In Figure 51 the reactions are shown as green vectors and the position of the support with red dots. By knowing the value of the reactions and their position and combining these forces with the external loading the trust line is obtained.

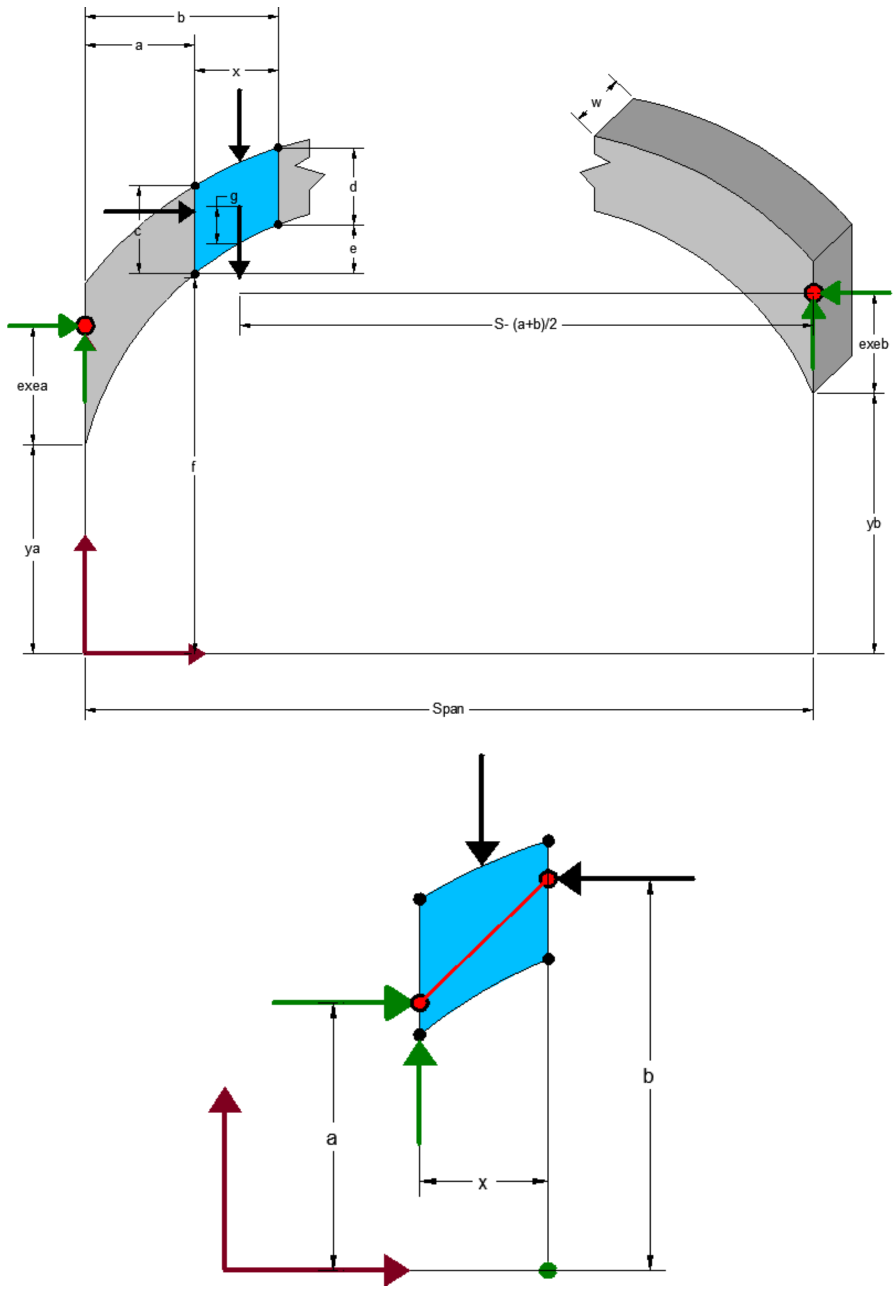

Figure 51: Free body diagram of the forces and reactions acting on the arch

Three unknowns, infinite thrusts of lines and the safety condition

The aim of this procedure is to find a horizontal thrust capable of building up a thrust line within the arch thickness. This reaction component and the position of the supports will not only be used to solved the indeterminacy of the system, but also assess the safety of the arch. Once the thrust at the supports is known the stability of the abutments can be assessed as well.

Thrust A	91
Position A	0.2
Position C	0.1
Status	Not safe

Figure 52: Parameters to be defined in the spreadsheet to build up the thrust line

The iterative procedure in finding a safety condition in the arch by changing the parameters described in Figure 52 is intuitive, and by graphically understanding the behavior of the arch, the safety condition in case exists can be founded easily.

Since the parameters showed in Figure 52 can be infinite, the number of thrust line will be infinite as well; the search of the conditions which produces at least one thrust line inside the thickness of the arch will be enough to guarantee the safety of the arch.

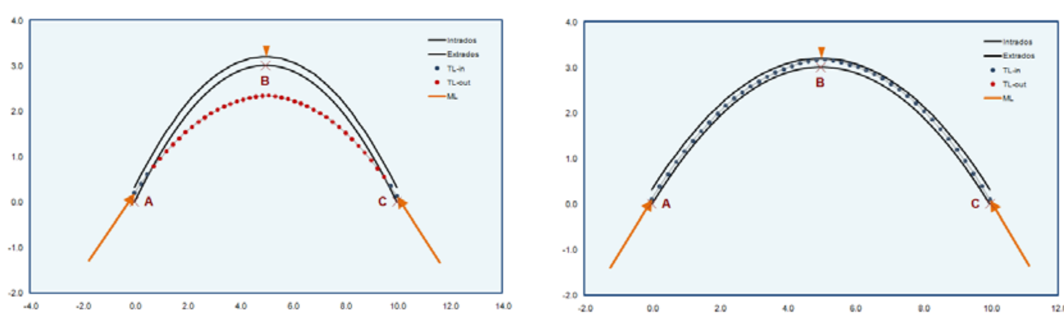

Figure 53: Thrust line found inside an arch after reducing the horizontal thrust

Loading criteria

Depending on the loading criteria, loads have to be applied on the arch, before start assuming the three variables discussed above. The spreadsheet will compute the self-weight automatically their accuracy will depend on the number of voussoirs considered for the analysis. Punctual loads and distributed dead and live loads can be also introduced manually. The loads can be vertical or horizontal, depending on the arches network.

Safety Factor

If at least one trust line can be found within the arch thickness, a safety condition of the arch can be guaranteed by the lower bound theorem. The safety factor which is a Live Load amplifier can be obtained by increasing Lambda when a safety condition can be found.

Once a thrust line cannot be found inside the arch thickness for a certain Lambda factor then the iterative procedure ends and Lambda can be determined. This Lambda values will give us an idea of the collapse mechanism of the arch, and the position of the hinges developed before the collapse.

A flow chart describing this algorithm is presented below in **Figure 56**. The algorithm described in this flow-chart is based on a trial an error iterative procedure which look for all the possible thrust lines that can be obtained by changing the horizontal thrust and the position of the supports.

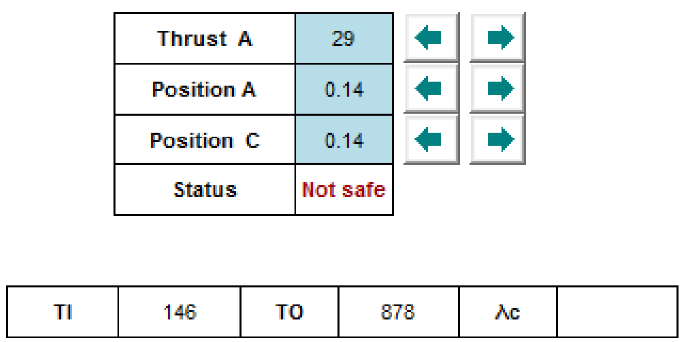

Figure 54: Parameters to be changed in the algorithm which maximizes Lambda

The algorithm described find the number of thrust lines within or outside the thickness of the arch, evaluating every single possible solution defined in *Figure 54,* this procedure is expensive the amount of data to be analyzed will depend on the horizontal forces range of application, and the steps into consideration for the parameters to be changed.

26.6	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14
0	In	Out	Out	Out	Out	Out	Out	Out
0.02	Out	Out	Out	Out	Out	Out	Out	Out
0.04	Out	Out	Out	Out	Out	Out	Out	Out
0.06	Out	Out	Out	Out	Out	Out	Out	Out
0.08	Out	Out	Out	Out	Out	Out	Out	Out
0.1	Out	Out	Out	Out	Out	Out	Out	Out
0.12	Out	Out	Out	Out	Out	Out	Out	Out
0.14	Out	Out	Out	Out	Out	Out	Out	Out

27.8	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14
0	Out	Out	Out	Out	Out	In	In	Out
0.02	Out	Out	Out	Out	In	In	Out	Out
0.04	Out	Out	Out	In	In	In	Out	Out
0.06	Out	Out	In	In	In	In	Out	Out
0.08	Out	In	In	In	In	In	Out	Out
0.1	In	In	In	In	In	In	Out	Out
0.12	In	Out	Out	Out	Out	Out	Out	Out
0.14	Out	Out	Out	Out	Out	Out	Out	Out

28.6	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14
0	Out	Out	Out	Out	Out	Out	Out	Out
0.02	Out	Out	Out	Out	Out	Out	Out	Out
0.04	Out	Out	Out	Out	Out	Out	Out	Out
0.06	Out	Out	Out	Out	Out	Out	Out	Out
0.08	Out	Out	Out	Out	Out	Out	Out	Out
0.1	Out	Out	Out	Out	Out	Out	Out	Out
0.12	Out	Out	Out	Out	Out	Out	Out	In
0.14	Out	Out	Out	Out	Out	Out	In	In

Figure 55: Output indicating if either the thrust line is encountered to be inside or outside depending on the support location and horizontal thrust magnitude

By running the algorithm tables as shown in *Figure 55* are generated, and it is possible to identify the minimum and maximum horizontal thrust from which a possible safe solution can be found. If at least one solution is found the Lambda coefficient can be increased if the maximization of Lambda is desired.

For our particular case since we are dealing with thin tile vaulted structures the range of application of the horizontal will be small, in the same way the position of the thrust line in the support will not vary much.

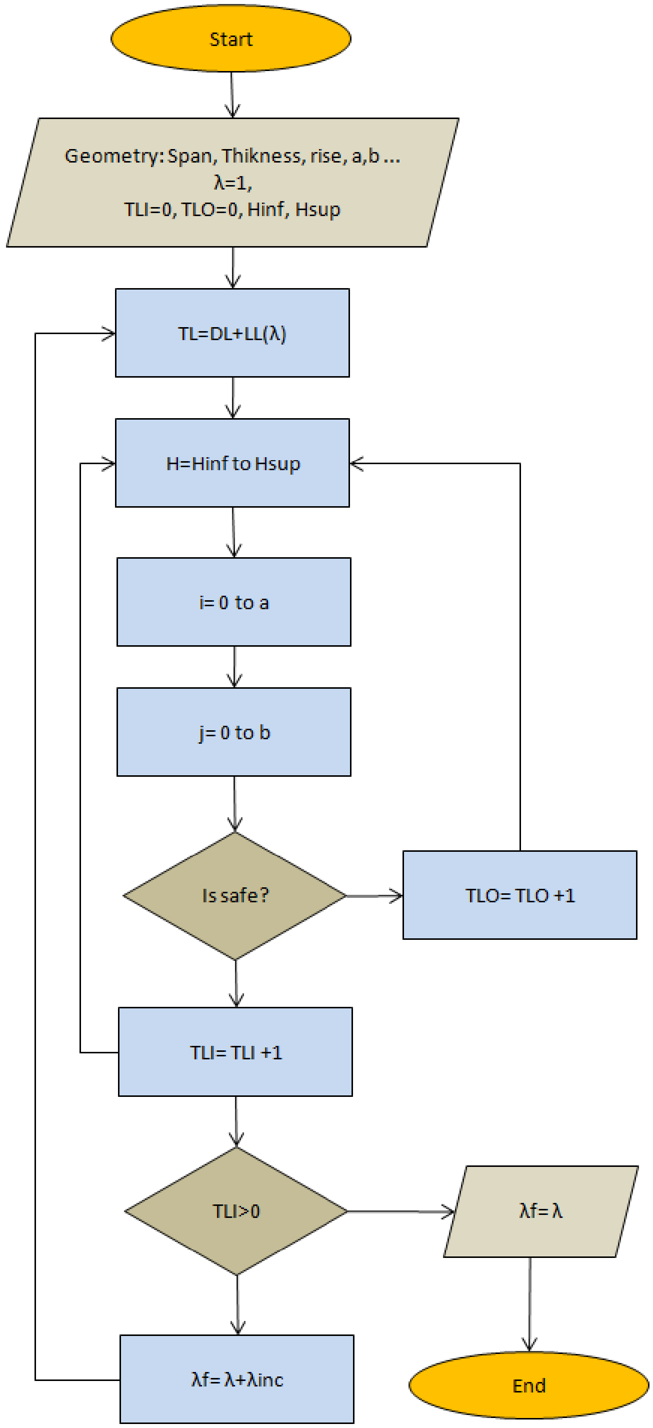

Figure 56: Flow-chart describing the general procedure to maximize Lambda

Spreadsheet validation

In order to validate the spreadsheet, a simple arch condition was graphically solved using static graphics in AutoCAD. Then once the geometry, load conditions and the assumed horizontal thrust were inputted into the spreadsheet a comparison of the reactions, thrust line and safety condition was carried out.

The validation was completed after the analysis results from the spreadsheet matched satisfactorily the results from the graphic analysis.

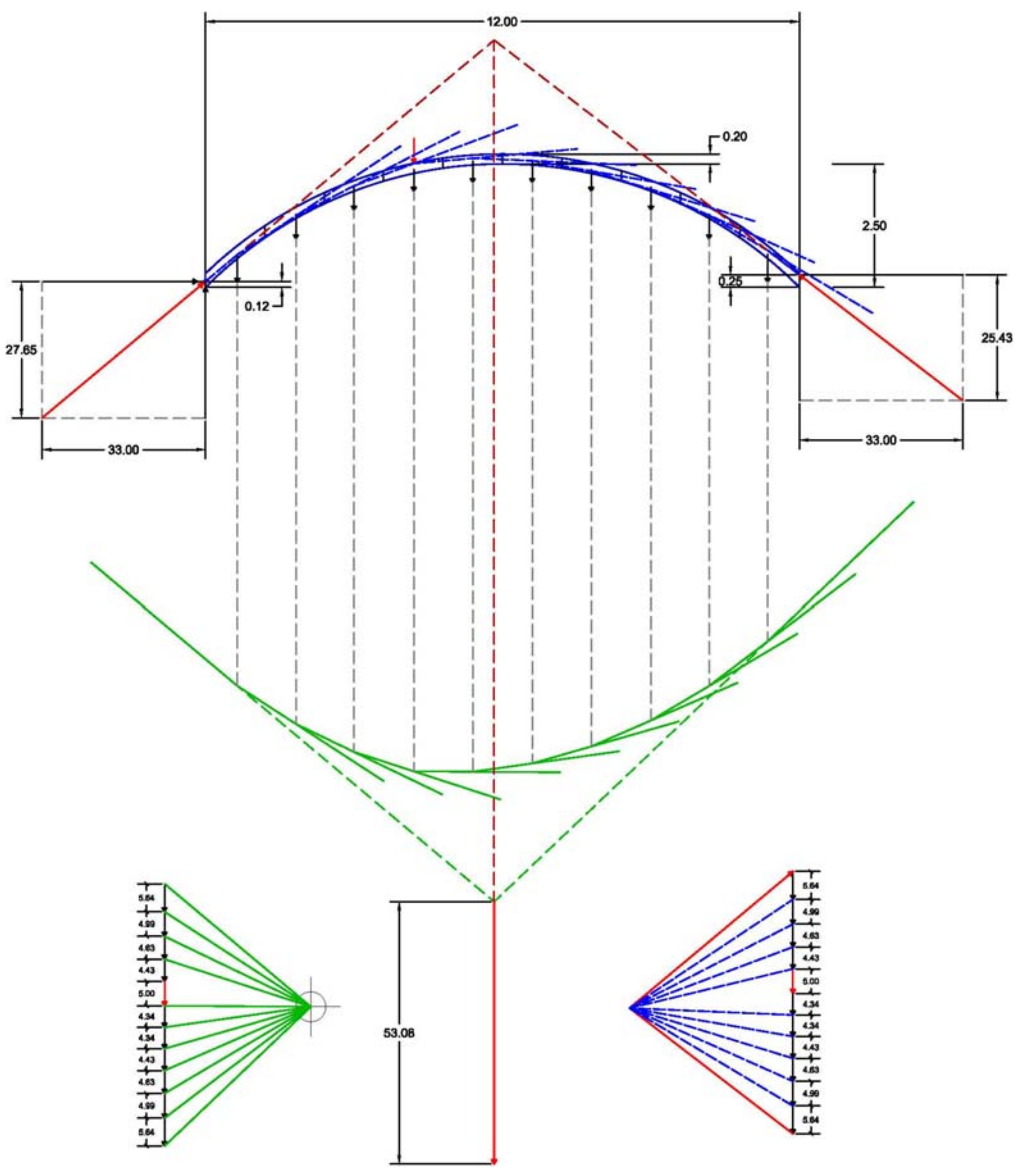

Figure 57: Solution of the validation problem using graphic statics

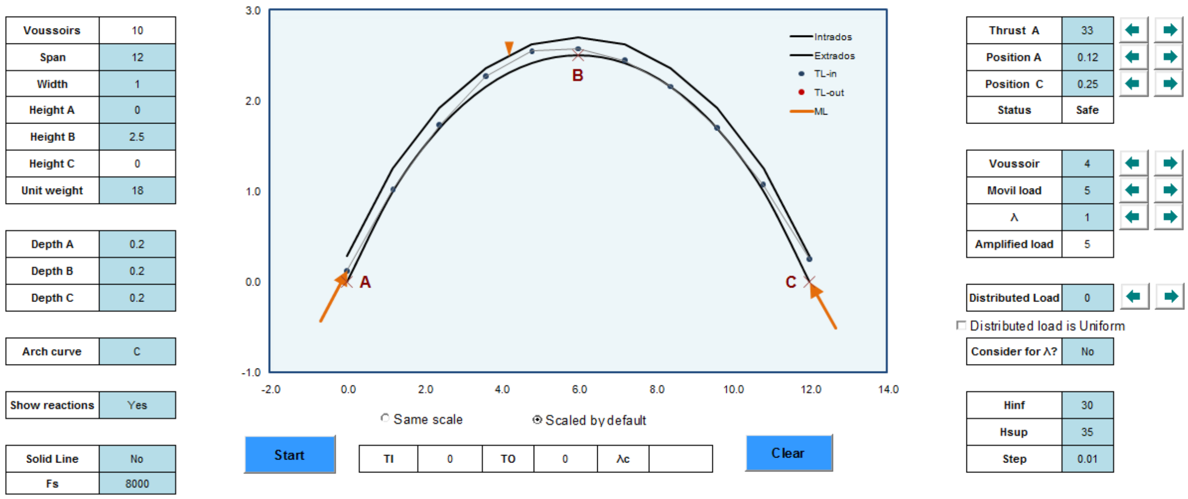

Figure 58: Spreadsheet showing the results of the validation exercise

APPENDIX B: LIVE LOAD COMPARISON

In the case of having found a thrust line inside the arches network and the safety of the vault guarantee, the process of maximizing Lambda which is the Live Load amplification factor will give a better insight on the safety of the vault by having an overall idea of the maximum magnitude and position of the live load which produces the instability of the vault.

A simple study was carried out with the aim of determining the position in which a punctual or distributed load produces the most unfavorable safety factor. The spreadsheet presented in Appendix A was used for the study.

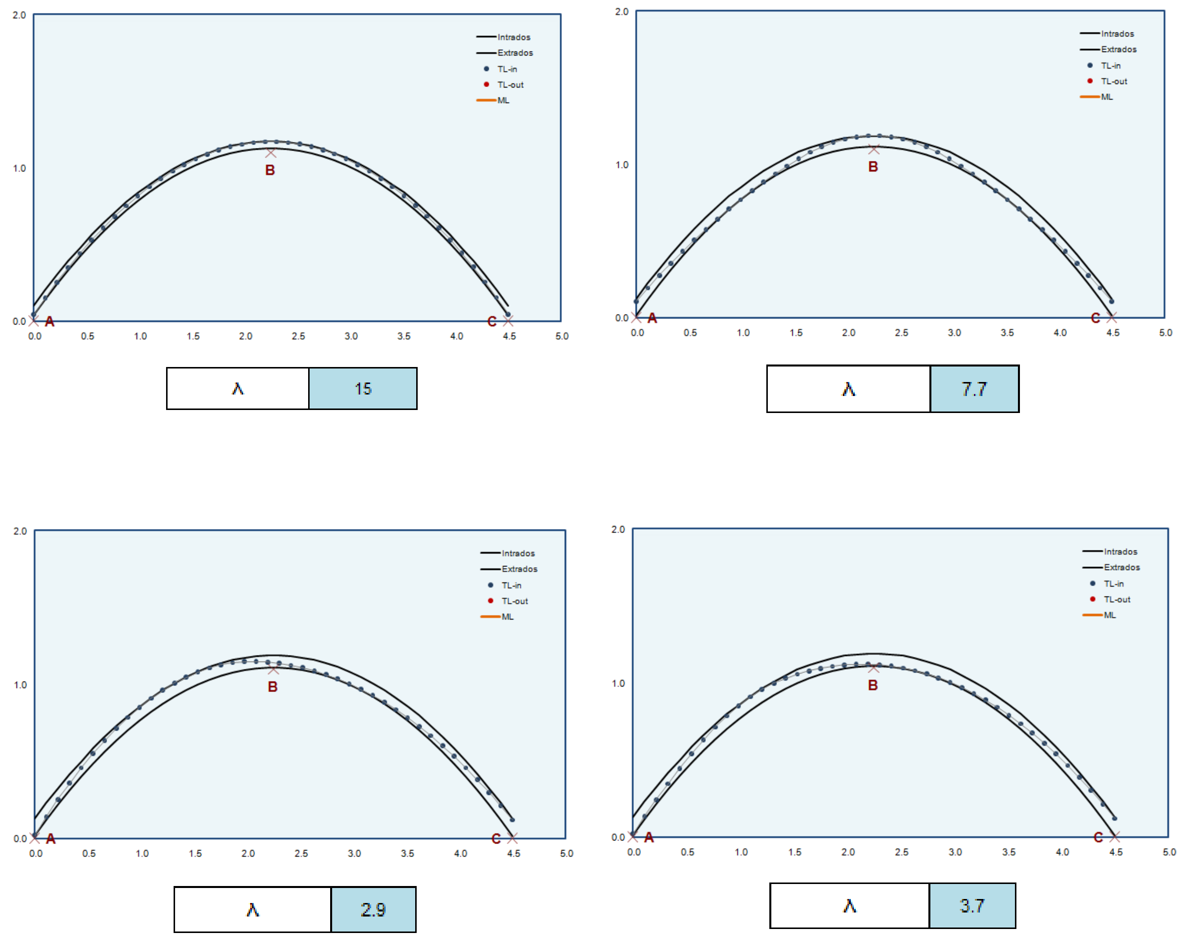

Figure 59 **:** Live load study

The study demonstrated that the loading condition which maximizes the instability of the arch happen when applying the live load uniformly distributed in half the span. In This loading condition a factor of 2.9 was reached.

The safer condition happens when the live load is applied uniformly over the arch; in this case scenario the assessment on the buttresses has to be evaluated in their ability to withstand the loads coming from the arch.

The point load applied at one quarter of the span of the arch showed also to produce low lambda coefficients.

This study was carried out with the intention of minimize the value of Lambda for certain models presented in the case studies, but the study is based on a 2D geometry and other studies should be considered for more complex models.

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